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RESEARCH OF POLYNOMIAL APPROXIMATION OF FORCES IN ROD ELEMENTS OF CORRODING STRUCTURES

Abstract. The paper considers the problem of polynomial approximation of the "force - time" dependencies in the elements of corroding hinged-rod structures from the point of view of the influence of the degree of the polynomial on the error in calculating their durability. A method for determining the coefficients of approximating polynomials is proposed, which is based on the use of a numerical-analytical algorithm for solving a system of differential equations describing the corrosion process. The results of numerical experiments are presented, illustrating for various constructions the dependence of the error in solving the problem on the degree of approximating polynomials.

Keywords: corrosion process, correction functions method, systems of differential equations, polynomial approximation.

Problem statement. Many structural elements of machines and units, primarily those used in the chemical and mechanical industries, are exposed during operations to the process environment that are corrosive to metal. This leads to its corrosive wear – the destruction of the subsurface layer of the metal, a change in the initial geometric characteristics of the structure, a decrease in the bearing capacity and, as a result, premature, often emergency, failure of the structural element. In the general case of corrosion interaction mechanical stresses accelerate the corrosion process.

The system of differential equations (SDE), which describes the corrosion process taking into account the influence of mechanical stresses, has the form:

$$\frac{d\delta_i}{dt} = v_0 \cdot \Phi\left(\sigma_i\left(\bar{\delta}\right)\right); \quad \delta_i\big|_{t=0} = 0; \; i = \overline{1,K}\,. \tag{1}$$

Here δ_i – is the depth of corrosion damage (damage parameter); v_0 – is the corrosion rate in the absence of stress; Φ – is the known stress function; t – is time; K – the number of damage parameters that uniquely determine the shape and dimensions of the structural element (SDE dimension).

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To calculate the stresses that are included in the right parts of the SDE (1), a system of equations of mechanics is used: equations of equilibrium and compatibility of deformations, Cauchy relations and physical relations (for elastic bodies – Hooke's law). Since the stress functions on the right side of (1) have the form of computational algorithm, only a numerical solution of system is possible (1).

When solving the problem of optimal design of corroding structures, the calculation of constraint functions (CF) involves the calculation of the stress state of the structure at a given point in time, taking into account the corrosion process occurring in it. The scheme for solving the optimization problem is a two-loop scheme (Fig. 1), where OF – is the module for calculating the objective function, CF – is the module for calculating the constraint function, SDE – is the module for solving the system of differential equations, SSS – is the module for solving the problem of mechanics, NLP – is the module for solving the problem of optimal design.

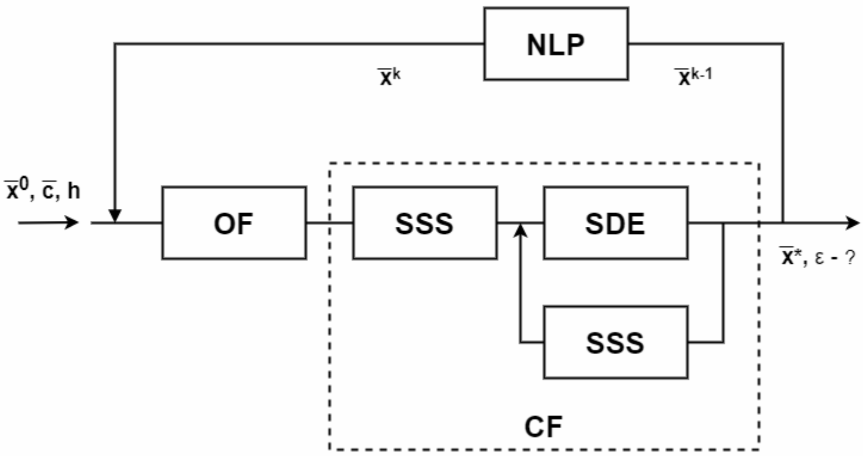

Figure 1 – Scheme for solving the optimization problem

The presence of feedback significantly increases the computational cost of solving the optimization problem and increases its sensitivity to errors that arise when calculating the constraint function.

Consider the error problem, which always arises when the parameter is the numerical solution of system (1) does not change in the process of solving the optimization problem. Let us assume that for some vector of variable parameters (VVP) $\bar{x}^s$ a numerical solution was obtained for the parameter *h* with an error ε^s, which is in the neighborhood of its maximum allowable value ε^*, set by customer: $\left|\varepsilon^s - \varepsilon^*\right| \le \delta$. For the new VVP $\bar{x}^{s+1}$ the error of the numerical solution ε^{s+1}, obtained with the same parameter *h*, will differ from ε^s.

Thus, the entire feasible set of solutions can be represented as the union of three subsets:

$$D = D_1 \cup D_2 \cup D_3. \tag{2}$$

Here D_1 – is a subset of solution whose error is higher than the maximum allowable, so any solution from this subset cannot be considered satisfactory; D_2 – a subset of solutions, the error of which is less than the maximum allowable; obtaining any of these solutions is inefficient in terms of computational costs; D_3 – a subset of solutions, the error of which lies in a certain neighborhood of the maximum allowable; any solution from this subset is optimal in terms of the efficiency of the numerical method.

Obviously, when using the traditional approach to solving the problem, the cardinality of the subset D_3 is incomparably less, than that of the other two. The ratio of the cardinalities of the subsets D_1 и D_2 depends on the choice of the parameter of the numerical solution of the SDE (1).

Increasing the accuracy and efficiency of numerical methods for solving the problems of calculating durability and optimal design of corroding structures involves increasing the cardinality of a subset D_3 in the ideal case – to the cardinality of the entire feasible set of solutions. To solve this problem, new approaches to modeling the process of corrosion deformation are needed, using methods of computational intelligence [1]. When building such models, the problem of polynomial approximation becomes very relevant.

Analysis of recent research and publications. In [2], apparently for the first time, an algorithm was proposed for controlling the accuracy of a numerical solution of an SDE of the form (1) using neural networks (NNs). It assumed the presence of several NNs, each of which made it possible to determine the parameter of the numerical solution for a given vector of variable parameters for a specific value of the error margin. Later, this algorithm was modified – is used a single NN, one of the input values of which was the value of the error margin [3]. The disadvantage of these algorithms was ignoring the change in internal forces in the elements of corroding structures when obtaining a training sample, as a result of which the real value of the error in solving the problem did not always coincide with the predicted one.

In [4], a correction functions method was proposed, in which the correction function (the error of the approximate solution of the SDE) was approximated by a neural network. Its input values were, among other things, the coefficients of the

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polynomial describing the change in time of internal forces in the elements of the hinged rod structure. The polynomial coefficients were determined at the stage of obtaining an approximate solution. Thus, the accuracy of solving the problem as a whole depended on the accuracy of the polynomial approximation.

Research objective. This paper proposes a research of the influence of the degree of polynomials approximating the dependence «force – time» in the elements of corroding hinged-rod structures, and the method of constructing them on the accuracy of calculating durability.

Presentation of the research main material. The solution of the problem of durability of a corroding hinged-rod structure (HRS) will be sought in the form:

$$t^*\left(\bar{x},\bar{y},\bar{c},h\right)=\tilde{t}\left(\bar{x},\bar{y},\bar{c}\right)\cdot\varphi_k\left(\bar{y},\bar{c},\bar{\alpha}\right);\ \ k=\overline{1,K}, \tag{3}$$

where $\tilde{t}\left(\bar{x},\bar{y},\bar{c}\right)$ – approximate solution obtained with minimal computational costs, $\varphi_k\left(\bar{y},\bar{c},\bar{\alpha}\right)$ – correction function, which is the error of the approximate solution; $\bar{x}$ – vector of variable construction parameters; $\bar{y}$ – vector of constant design parameters; $\bar{c}$ – vector of process environment parameters; $\bar{\alpha}$ – vector of parameters determined in the process of obtaining an approximate solution; h – parameter of the numerical solution of the SDE; K – the number of correction functions required to solve the problem.

In our case $\bar{\alpha}$ is a vector of polynomial coefficients approximating the dependence of internal forces in the rod elements of the structure on time. It is obvious that the degree of the approximating polynomial and the way it is constructed will significantly affect the value of the correction function and the accuracy of solving the problem as a whole. In this case, the values of the coefficients must be determined when implementing the algorithm for obtaining an approximate solution to the durability problem.

To obtain an approximate solution, it is proposed to use a numerical-analytical method for solving SDEs of the form (1), which is quite fully described in [5].

For the corroding element of the HRC, an analytical formula is known that allows you to determine the time during which the stress in it increases from σ_0 to σ at a constant value of axial force Q:

$$t^*_{\text{an}}=t_0-\frac{2kQ}{v_0\cdot|d|}\left\{arctg\frac{2a\delta-P_0}{|d|}+arctg\frac{P_0}{|d|}\right\}, \tag{4}$$

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Here A_0, P_0 – area and perimeter of the section at the initial moment of time; k – coefficient of influence of stress on corrosion rate; $t_0 = \dfrac{\delta^*}{v_0}$; a – section shape factor; $c = A_0 + kQ$; $d = \sqrt{\left|P_0^{\,2} - 4ac\right|}$, $(d \neq 0)$; δ^* – depth of corrosion wear corresponding to the limit state of the element.

In well-known numerical-analytical algorithms, it is proposed to use a uniform step by stress $\Delta\sigma = \dfrac{[\sigma] - \sigma_0}{n} = const$ (Fig. 2), and the corresponding value Δt determine by formula (4).

It is obvious that the accuracy of the numerical-analytical algorithm will depend on the number of splitting points of the interval $(\sigma_0;[\sigma])$.

A 5-rod statically indeterminate HRC was taken as a model design (Fig. 3). The parameters of the HRC and the process environment were assumed to be known: L = 200 cm; E = 2,1×10^5 MPa; $[\sigma]$ = 240 MPa; v_0 = 0,1 cm/year, k = 0,003 MPa^{-1}; external load value P = 250 kN.

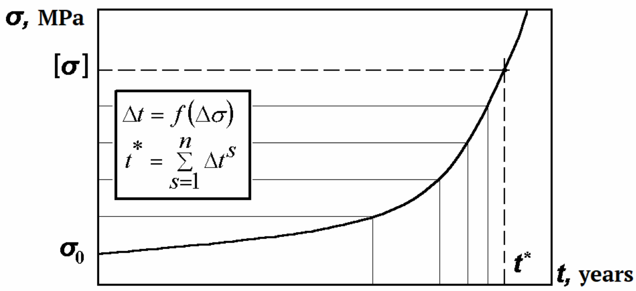

Figure 2 – Graphic illustration of the method

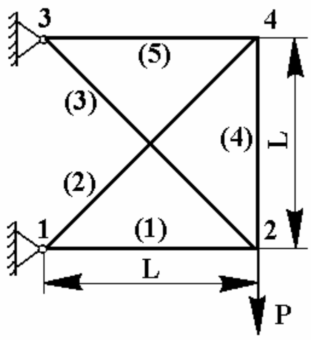

Figure 3 – Design scheme HRC

All rods had sections corresponding to standard shaped profiles (channel, I-beam, equal and unequal angles) of various standard sizes. Numerical experiments were carried out with various parameters of the rods, which were chosen randomly. For each option, a reference solution of the durability problem was obtained. In this case, system (1) was solved numerically using the Runge-Kutta method of the 2nd order of accuracy. The numerical solution parameter was taken equal to

$$\Delta t = 0{,}005 \cdot t^*,$$

where $t^* = \min\{t^*_1, t^*_2, t^*_3, t^*_4, t^*_5\}$ – element durability value calculated from the formula (4) and determining the durability of the structure as a whole. Internal

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forces and stresses in HRC elements were determined by the finite element method (FEM).

Two HRCs were selected for further research, differing only in the parameters of the rod elements. Their durability was determined by the durability of the elements: element (2), working in compression (construction (A), $t_A{}^* = 2{,}688$ year), and элементом element (3), working in tension (construction (B), $t_B{}^* = 2{,}126$ year). Changes in internal forces over time in these elements are shown in Fig. 4 (solid and dashed lines, respectively). For clarity, the values of time and force are given in dimensionless quantities:

$$\tilde{t} = \frac{t}{t^*}; \; \tilde{Q} = \frac{Q - Q_0}{Q_t - Q_0}. \tag{5}$$

Here $Q_0 = Q\big|_{t=0}; \; Q_t = Q\big|_{t=t^*}$.

The choice of structures was determined by the nature of the changes in the forces. For construction (A) function graph $Q_A(t)$ has points of maximum and inflection; for construction (B) it is a smooth curve. Obviously, the polynomial approximation $Q_A(t)$ is a more difficult problem than the approximation $Q_B(t)$.

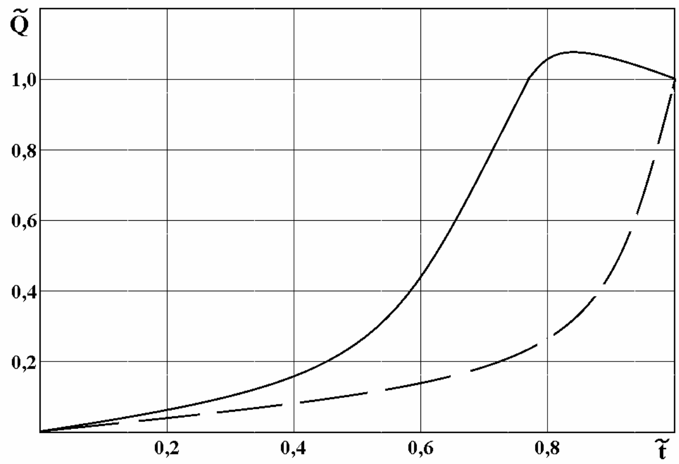

Figure 4 – Change in internal forces over time

Formula (4) makes it possible to obtain an exact solution of the durability problem only in the case when the internal forces in the rod elements of the structure remain constant throughout the entire period of its operation, i.e. for a statically defined construct. On the other hand, if the element number is known *m*, the durability of which determines the durability of the structure as a whole, and the law of change in the force in it, then the solution of a single differential equation of the form:

$$\frac{d\delta_m}{dt} = v_0 \cdot \Phi\left(\frac{Q_m(t)}{A_m^0 - P_m^0\delta + a\delta^2}\right) \tag{6}$$

will coincide with the solution of the system (1).

Let approximate the dependence $Q_m(t)$ by a polynomial of degree *n*:

$$Q_m(t) = P_n(t) = \sum_{k=0}^{n} \alpha_k \cdot t^k . \tag{7}$$

To determine the coefficients of the polynomial, information is needed on the stress-strain state of the HRC at (n+1) nodal points, including the initial time $t = 0$. The application of the numerical-analytical method will make it possible to obtain ab approximate solution $\tilde{t}(\bar{x}, \bar{y}, \bar{c})$ with minimal computational costs (the number of calls to the FEM procedure is determined by the order of the polynomial (7)) and, at the same time, determine the number of the element that determines the durability of the structure and the values of the polynomial coefficients.

Thus, the method for calculating the coefficients of polynomial (7) is determined by the method of obtaining an approximate solution. Next, it is necessary to determine the degree of the polynomial, which provides the required accuracy of the calculation.

When obtaining reference solutions to the durability problem, tabular values of the functions $Q_A(t)$ and $Q_B(t)$ were obtained. We will use these functions in the numerical solution of the differential equation (6). In this case, the results will coincide with the corresponding reference solutions of the durability problem for structures (A) and (B). If approximating polynomials (7) are used in the numerical solution of (6), then the divergence between the results will depend only on their degree. As a criterion for the quality of the polynomial approximation, we take the relative error of the numerical solution (6) using (7).

Some research results are shown in the Table. 1.

Table 1

$P_n(t)$	Construction (A) $t_A{*} = 2{,}688$ year		Construction (B) $t_B{*} = 2{,}126$ year	
	t_n, years	ε, %	t_n, years	ε, %
2	2.871	6,809	2.215	4,191
3	2.775	3,244	2.148	1,032
4	2.727	1,434	2.239	0,533
5	2.713	0,922	2.209	0,391

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As follows from the above results, an increase in the degree of approximating polynomials does not lead to a significant increase in the accuracy of calculations.

Conclusions. The study made it possible to substantiate the parameters of polynomial approximation when using the method of correction functions for solving problems of durability and optimal design of corroding hinge-rod systems. Since the correction functions are neural networks, the presence of several matrices of synaptic weights obtained for polynomials of different degrees will allow you to control the accuracy of calculating the constraint functions in the process of solving the optimization problem. This approach is consistent with the strategy of the sliding tolerance method and can significantly reduce computational costs**.**

ЛІТЕРАТУРА

1. Зеленцов Д.Г., Короткая Л.И. Технологии вычислительного интеллекта в задачах моделирования динамических систем. Днепр: Баланс-Клуб, 2018. – 178 с.
2. Зеленцов Д.Г., Короткая Л.И. Использование нейронных сетей при решении задач долговечности корродирующих конструкций. // Вісник Кременчуцького національного університету ім. М.Остроградського. – Кременчук: КрНУ, 2011. – Вип. 3 (68), част. 1. – С. 24 – 27.
3. Денисюк, О. Р. Определение рациональных параметров численного решения систем дифференциальных уравнений // Вестник Херсонского национального технического университета. – 2016. – №3(58). – С. 208 – 212.
4. Zelentsov D.G., Denysiuk O.R., Korotka L.I. The Method of Correction Functions in Problems of Optimization of Corroding Structures. // Advances in Computer Science for Engineering and Education III (ICCSEEA 2020), 2020. – pp. 132 – 142.
5. Зеленцов Д.Г., Науменко Н.Ю., Ляшенко О.А. Информационное обеспечение расчётов корродирующих объектов. Конечно-элементное моделирование. Днепр: Баланс-Клуб, 2018. – 174 с

REFERENCES

1. Zelentsov D.G., Korotkaya L.I. Tehnologii vyichislitelnogo intellekta v zadachah modelirovaniya dinamicheskih sistem. Dnepr: Balans-Klub, 2018. – 178.
2. Zelentsov D.G., Korotkaya L.I. Ispolzovanie neyronnyih setey pri reshenii zadach dolgovechnosti korrodiruyuschih konstruktsiy. // Visnyk Kremenchutskoho natsionalnoho universytetu im. M.Ostrohradskoho. – Kremenchuk: KrNU, 2011. – Vyp. 3 (68), chast. 1. – S. 24 – 27.
3. Denisyuk, O. R. Opredelenie ratsionalnyih parametrov chislennogo resheniya sistem differentsialnyih uravneniy // Vestnik Hersonskogo natsionalnogo tehnicheskogo universiteta. – 2016. – №3(58). – С. 208 – 212.

4. Zelentsov D.G., Denysiuk O.R., Korotka L.I. The Method of Correction Functions in Problems of Optimization of Corroding Structures. // Advances in Computer Science for Engineering and Education III (ICCSEEA 2020), 2020. – pp. 132 – 142.
5. Zelentsov D.G., Naumenko N.Yu., Lyashenko O.A. Informatsionnoe obespechenie raschYotov korrodiruyuschih ob'ektov. Konechno-elementnoe modelirovanie. Dnepr: Balans-Klub, 2018. – 174 s.
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Дослідження поліноміальної апроксимації зусиль в стержневих елементах кородуючих конструкцій

У роботі розглядається проблема поліноміальної апроксимації залежностей «зусилля – час» в елементах шарнірно-стержневих конструкцій з точки зору впливу ступеня поліному на похибку обчислення їх довговічності. Процес корозії в конструкціях описується системою диференційних рівнянь із врахуванням механічних напружень, яка може бути розв'язана тільки з використанням чисельних методів. Алгоритм розв'язання задачі оптимального проектування кородуючих конструкцій вимагає обчислення функцій обмежень, які передбачають обчислення напруженого стану конструкції в певний момент часу. Наявність зворотного зв'язку в даному алгоритмі збільшує обчислювальні витрати та чутливість до похибок. Останні дослідження пропонують використання нейронних мереж для контролю точності чисельних розв'язків систем диференційних рівнянь, однак ці алгоритми ігнорують зміну внутрішніх зусиль у кородуючих конструкціях. Пізніше був запропонований метод коригувальних функцій, де коригувальна функція була апроксимована нейронною мережею з використанням коефіцієнтів полінома, який описує зміни внутрішніх зусиль з часом. Коефіцієнти поліному визначалися на етапі отримання наближеного розв'язку, таким чином точність розв'язку задачі в цілому залежала від точності поліноміальної апроксимації. Метою роботи є дослідження впливу ступеня поліномів, які апроксимують залежність «зусилля – час» в елементах кородуючих конструкцій, та способу їх отримання на точність обчислення довговічності. В роботі запропоновано спосіб визначення коефіцієнтів апроксимуючих поліномів, який ґрунтується на використанні чисельно-аналітичного алгоритму розв'язання системи диференційних рівнянь, що описують корозійний процес. Наведено результати чисельних експериментів, що ілюструють для різних конструкцій залежність похибки розв'язку задачі від ступеня апроксимуючих поліномів. Наведений підхід узгоджується зі стратегією нежорсткого допуску і дозволяє суттєво знизити обчислювальні витрати.

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