

O.P. Sarychev, B.A. Perviy

## **AUTOREGRESSION MODELS OF LARGE SPACE DEBRIS MOTION**

*A statistical method was developed for modeling the large space debris motion in the class of autoregressive models. The method improves the quality of description and forecasting of the movement of large fragments of space debris based on their TLE elements.*

*Keywords: TLE elements, structural uncertainty, beta distribution, unequally spaced observations.*

### **Introduction**

Technogenic pollution on low earth orbits level is an urgent problem of modern astronautics. One of the debris segments is large fragments of space debris: firstly, these are spacecraft that have ceased to exist (non-functioning), and, secondly, these are the last stages of launch vehicles. The development of an effective method for the removal of large fragments of space debris into low orbits is an important scientific and technical problem. One of the ways to deal with large fragments of space debris is a non-contact (without mechanical capture) impact in order to carry the fragments to lower orbits for further reduction due to aerodynamic braking. The implementation of this method implies finding the solution for two problems: the first task is to construct mathematical models of space debris motion based on the results of their observation; the second task is to select a method of impact for the effective removal of space debris fragments into low orbits.

The aim of this work is to improve the accuracy of modeling the large space debris motion using their TLE elements.

### **Major part**

*1 A method for modeling of large space debris motion in the class of autoregressive models using their TLE elements*

Determination of the order and estimation of the coefficients of autoregressive models in conditions of structural uncertainty by the number and composition of regressors is an urgent problem of the theory of identification, and there are various approaches to its solution. In the overwhelming majority of cases, the existing identification methods do not assume that in the autoregressive models, certain rela-

tions are satisfied between the coefficients at the previous values of the variables. In our opinion, such ratios are natural: previous values close in time should have coefficients close to each other in the model, and previous values that are more distant in time should have less close ones. When choosing one or another relationship between the autoregression coefficients, it is necessary to take into account the "physics" of the object, the available a priori information, as well as the discreteness of observations of the state of the object in time. Examples of this approach are moving average, exponential smoothing, and other windowed methods. In this paper, it is proposed to search for the optimal smoothing window in the family of probability density functions of beta distributions [1].

A distinctive feature of time series of TLE-elements is their representation by observations not on a uniform grid in time, but at irregular intervals - the so-called. "unequally spaced observations". This distinctive feature of the TLE-element time series is used to modify the parameter estimation procedures when constructing autoregressive models of large space debris motion [1]. To model this kind of time series, an iterative procedure for the parametric identification of autoregressive models with unequally spaced observations was developed, the effectiveness of which was confirmed by the method of statistical tests.

The construction of autoregressive models of the of large space debris motion using their TLE-elements, obtained under conditions of unequally spaced observations, is carried out in a modeling system developed on the basis of the results of [1] - [6].

Time series of TLE-elements [7] are represented by seven basic and three additional variables (see Table 1).

Simulations were carried out for six main variables, since the variable  $x_4$  (inclination) is constant. Additional variables  $x_8$ ,  $x_9$  were used to construct figures, and the variable  $x_{10}$  was used to calculate the power to which the components of the coefficients are raised in the iterative procedure for parametric identification of autoregressive models in conditions of unequally spaced observations [1] - [2].

In accordance with the developed method of structural-parametric identification in [1] - [2], the ratios of the weight coefficients for autoregressors in the beta autoregressive model were set by a special weight function determined on the basis of the two-parameter probability density function of the beta distribution [8], in which the parameter  $\alpha$  is fixed -  $\alpha = 1$ , and the parameter  $\beta$  can be integer values  $\beta = 1, 2, \dots, \beta_{\max}$ , where  $\beta_{\max}$  is the specified value.

List of variables for Sich– 2 TLE data

Designation	Title	Unit of measurement
$x_1$	Apogee	km
$x_2$	Perigee	km
$x_3$	Eccentricity	–
$x_4$	Inclination	deg
$x_5$	Right ascension of the ascending node	deg
$x_6$	Argument of perigee	deg
$x_7$	Mean anomaly	deg
$x_8$	Revolution number at epoch	revs
$x_9, t_{nak}$	Accumulated time	hrs
$x_{10}, \tau_i$	The time interval between the current and the previous observation	hrs

The larger the parameter value  $\beta$ , the greater the excess of the weight coefficient at the first previous value of the variable over the coefficients at the earlier previous values. The graphs of weighting functions for  $\beta = 1, 3, \dots, 25$  with  $p = 7$  are shown in Fig. 1 (the weight functions are presented as the values of the probability density functions for a random variable having a beta distribution).

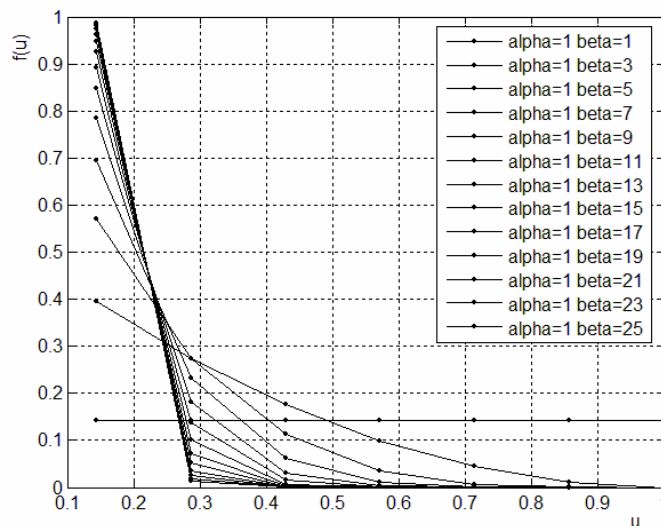


Figure 1 –Weight functions of coefficients for beta autoregression ( $p = 7$ )

Having  $\beta = 1$ , the weighting coefficients for autoregressors are the same:  $a_j = 1/7, j = 1, 2, \dots, 7$ . When  $\beta$  is increasing, the weighting factor at the first previ-

ous value becomes dominant. So, for  $\beta = 13$ , the weight of the first "delay" in the model is approximately 0.90, and the sum of the weights of all other delays (their number is equal  $p - 1 = 6$ ) is approximately 0.10. Having  $\beta = 25$ , the weight of the first delay is approximately 0.99, and the sum of the weights of all other delays is approximately 0.01.

Having completed the construction of beta-autoregressive models for all  $\beta = 1, 3, \dots, 25$ , it is possible to identify the best structure (weight function, optimal parameter of value  $\beta$ ), which corresponds to the smallest mean-square error of the model.

By dividing the entire observation period into intervals corresponding to the cycles of changes in the main variables  $x_5$ ,  $x_6$  и  $x_7$ , and solving the problems of structural and parametric identification for all basic variables based on [1] - [2], it is possible to reveal a change in the structure (type of the weight function), coefficients and mean square error of the best model main variables depending on the interval number. By analyzing the graphs of changes in these characteristics, it is possible to identify the features and general patterns of their behavior for different objects during the observation period.

### *2 Large space debris motion modeling using autoregression*

In [2], the described modeling scheme is applied for a number of space objects. In this work, an attempt is made to detect more detailed changes in motion for a number of spacecraft and spent stages of launch vehicles. For this, for all objects, each of the cycles in the variable  $x_5$  and each of the cycles in the variables  $x_6$  and  $x_7$  (as well as  $x_1, x_2, x_3$ ) were additionally divided into several intervals so that the average length of the new intervals was about 100 values.

The simulation results for two spacecraft (SICH1 # 23657 and POLYITAN-1 # 40042 NanoUkr) and two spent stages (ATLAS 2A CENTAUR # 26353 and SL-6 RB (2) # 18086) are shown in Fig. 2 - 5, which shows the dependences of the mean square error of the constructed models of six main variables.

The generalized results of visual analysis Fig. 2 - fig. 5 are given in table. 2, which shows the ranges of variation of the mean square error of the constructed models, corresponding to the "normal mode" of their motion.

Root mean square errors of models for spacecraft and spent stages

Name	Ranges of RMS model errors					
	$x_1, m$	$x_2, m$	$x_3, \cdot$	$x_5, deg$	$x_6, deg$	$x_7, deg$
KA SICH1	3 – 10	0,5 – 10	0,5 – $1,0 \times 10^{-5}$	0,1 – 0,4	1 – 2	1 – 2
KA POLYITAN-1	0,5 – 2	0,5 – 2	0,2 – $0,4 \times 10^{-5}$	0,05– 0,1	0,2 – 0,7	0,2 – 0,7
ATLAS2A CENTAUR	100–800	100–800	$3 - 15 \times 10^{-5}$	0,1 – 0,8	0,2 – 1,0	0,2 – 2,0
SL-6 RB(2)	400–500	900– 5000	10 – $20 \times 10^{-5}$	0,04– 0,06	0,02– 0,06	0,02– 0,05

In certain time series intervals, the model errors take on values that significantly exceed the values from the ranges given. A detailed visual analysis of such intervals shows that they, as a rule, contain abrupt changes in the behavior of the main characteristics that are not characteristic of the usual movement of objects. These exceedances can be used as signs of an unforeseen change in the movement of a space object.

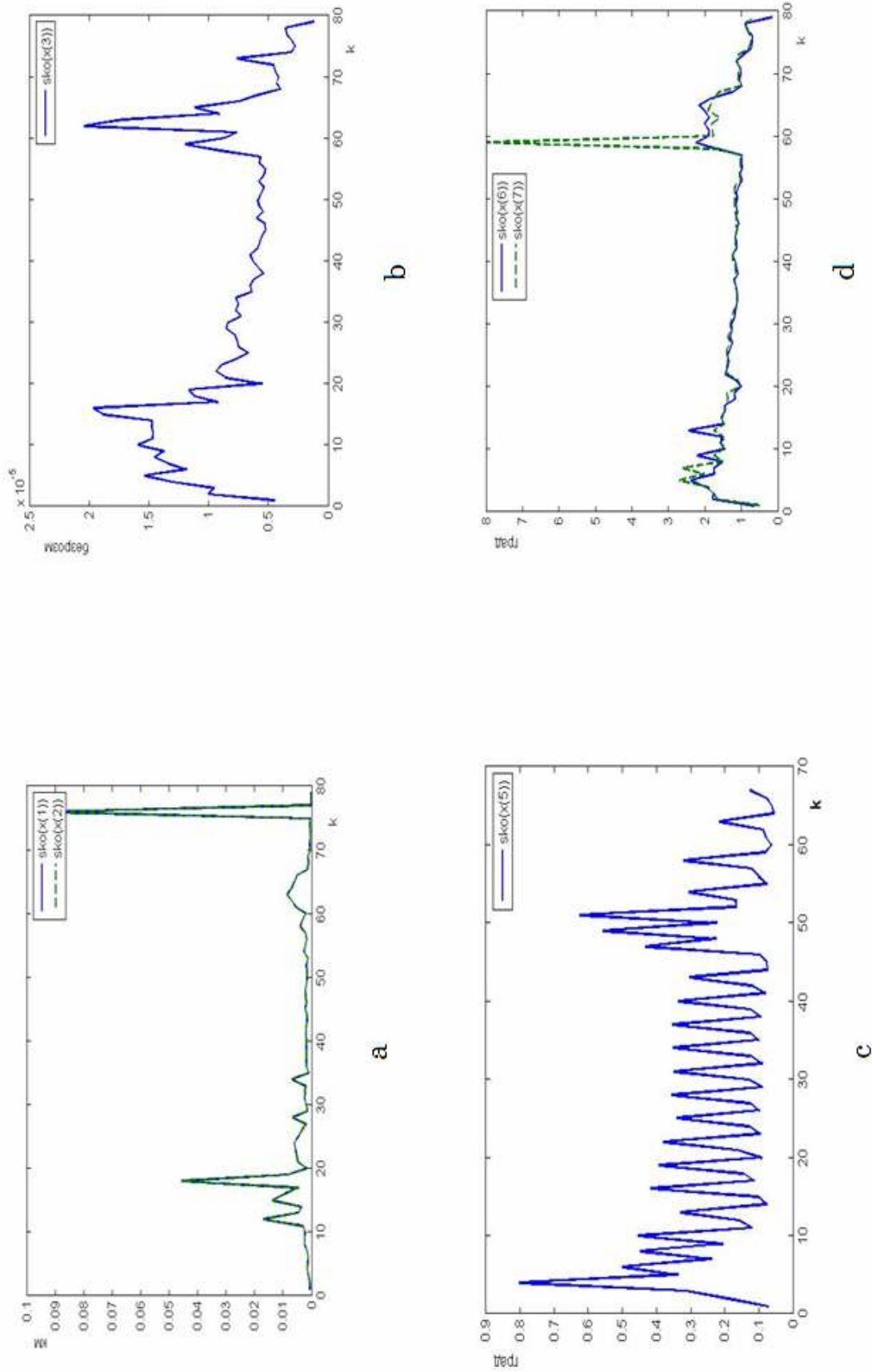


Figure 2 - RMS of the models for SICH1 #23657: a – apogee ( $x_1$ ), perigee ( $x_2$ ), b – eccentricity ( $x_3$ ), c – longitude of the ascending node ( $x_5$ ), d – argument of perigee ( $x_6$ ), mean anomaly ( $x_7$ )

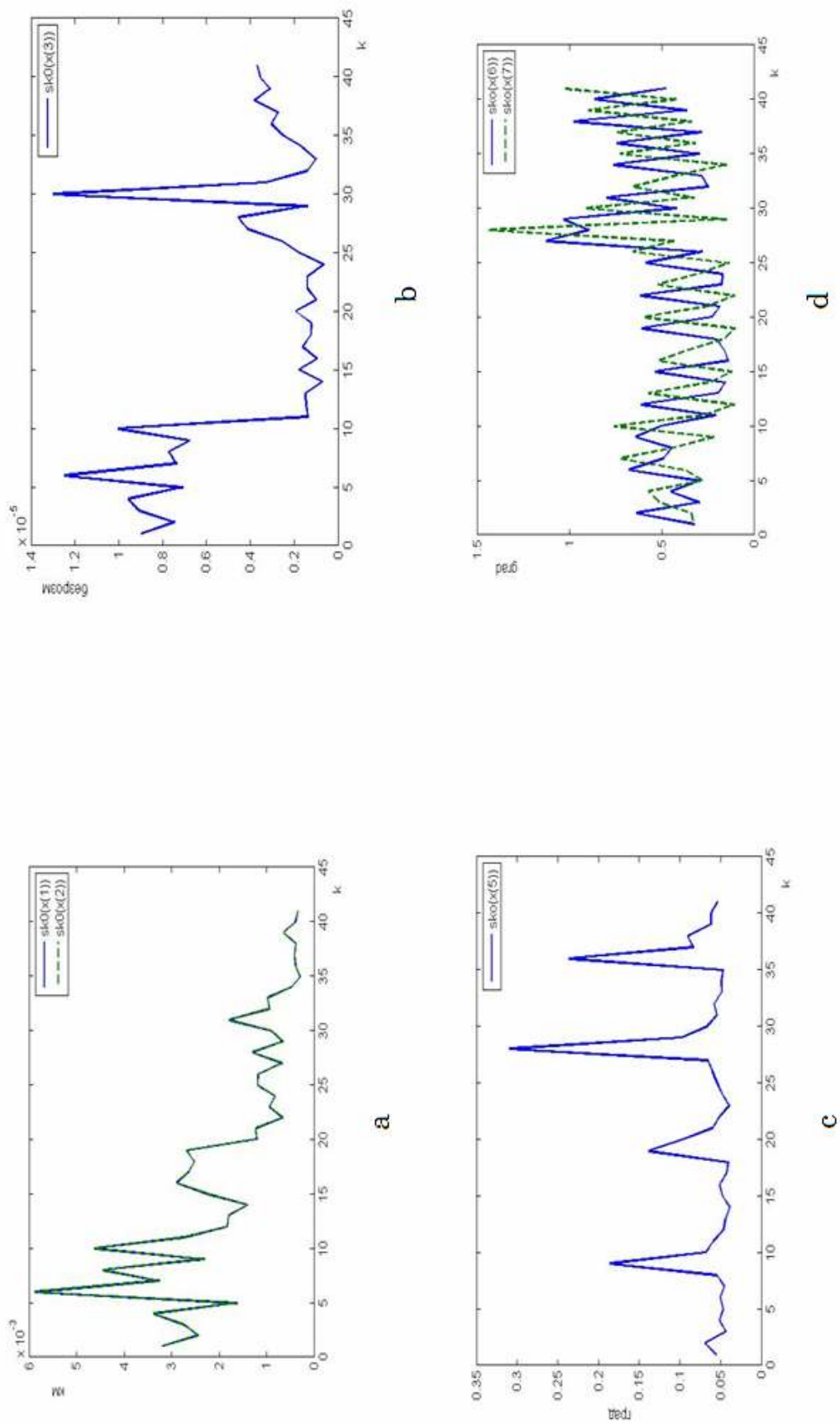
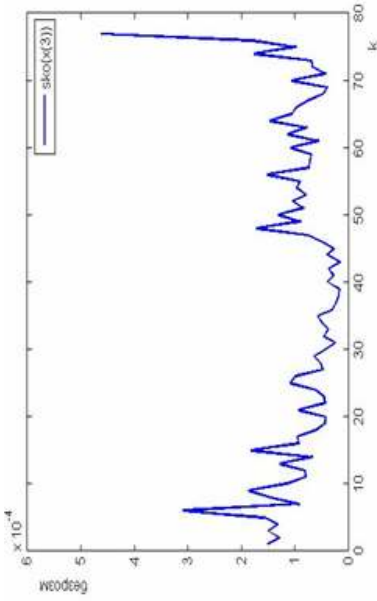
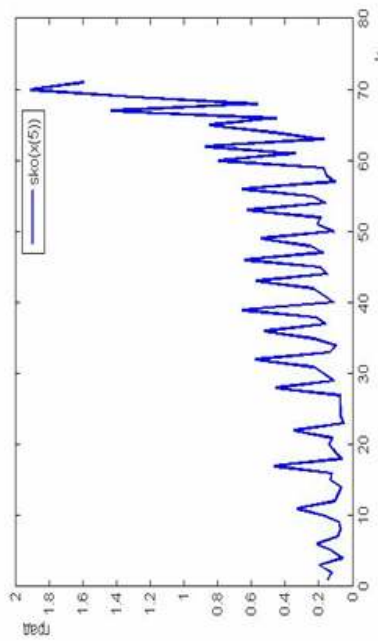


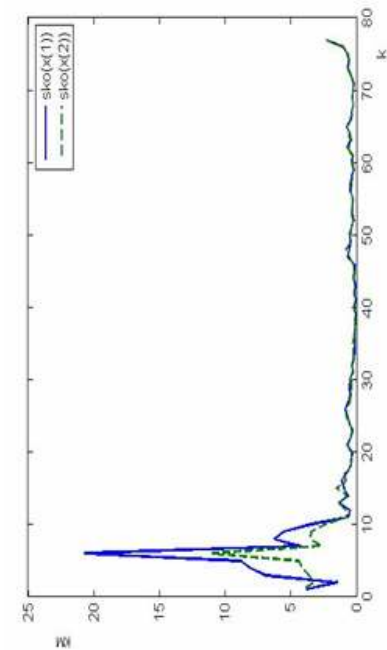
Figure 3 - RMS of the models for POLYITAN-1 #40042: a – apogee ( $x_1$ ), perigee ( $x_2$ ), b – eccentricity ( $x_3$ ), c – longitude of the ascending node ( $x_5$ ), d – argument of perigee ( $x_6$ ), mean anomaly ( $x_7$ )



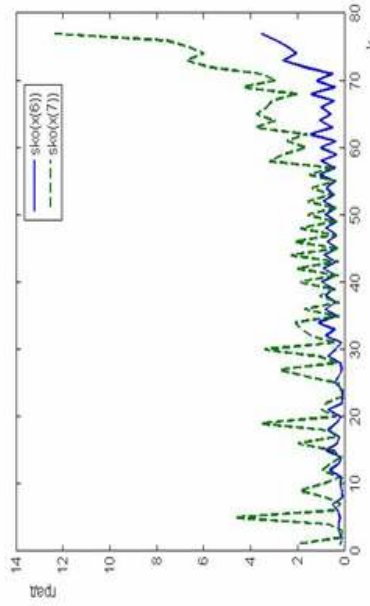
a



c



b



d

Figure 4 - RMS of the models for ATLAS\_2A\_CENTAUR\_#26353: a – apogee ( $x_1$ ), perigee ( $x_2$ ), b – eccentricity ( $x_3$ ), c – longitude of the ascending node ( $x_5$ ), d – argument of perigee ( $x_6$ ), mean anomaly ( $x_7$ )



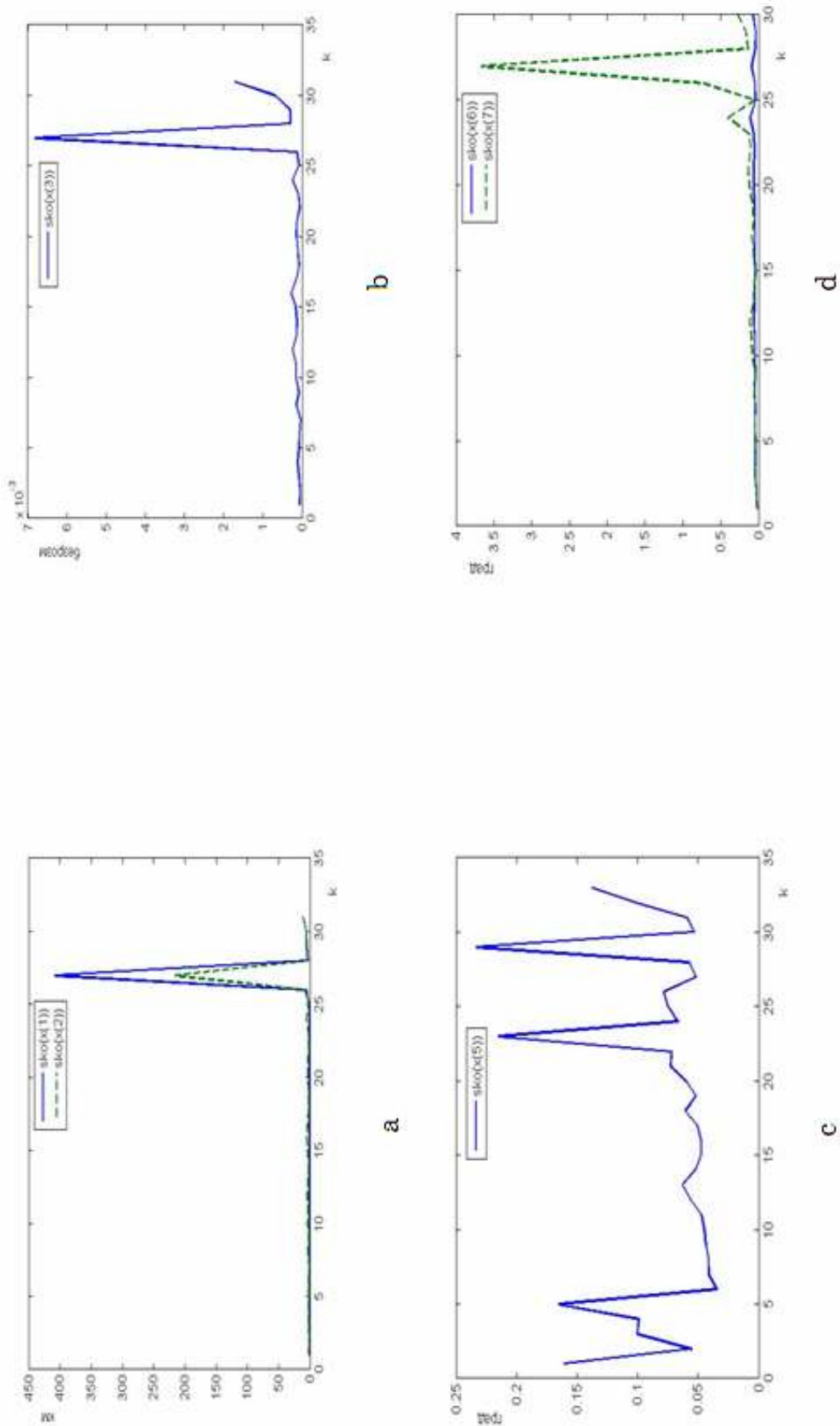


Figure 5 - RMS of the models for SL-6 RB(2) #18086: a – longitude of ascending node ( $x_5$ ), b – apogee ( $x_1$ ), c – perigee ( $x_2$ ), d – eccentricity ( $x_3$ ), mean anomaly ( $x_7$ )

### **Conclusion**

Based on the results obtained in the process of modeling the motion of two groups of large objects of space debris (spacecraft and spent stages of launch vehicles), represented by the time series of TLE elements, a modeling system has been developed, which includes:

- a) determination of the optimal volume of training samples when modeling time series of TLE elements;
- b) determining the order of autoregression for each variable element;
- c) determination of the optimal structure and identification of the parameters of the autoregression model for each variable element;
- d) establishing the features of the behavior of the mean square error of autoregressive models in time based on the modeling of time series of TLE elements;
- e) obtaining predictive estimates of the values of variable elements at future points in time.

The developed statistical method for modeling the movement of large fragments of space debris can be recommended for describing and predicting the movement of spacecraft and spent stages of launch vehicles, represented by time series of TLE elements, for solving an important scientific and technical problem - the development and study of effective methods for removing large fragments of space debris to low orbits for further reduction due to aerodynamic braking.

The use of the developed statistical modeling method will make it possible to control the process of removing a large object of space debris based on models built from the time series of its TLE elements, which are promptly updated and are in the public domain.

As a result, there are two main approaches to assessing TLE elements in the literature: machine-expensive methods for searching global optics and methods for searching local optics. The developed autoregressive models [18] provide alternative, less machine-costly methods for improving forecasts, and their expected errors do not exceed the error rates presented above the models.

### **ЛІТЕРАТУРА / ЛИТЕРАТУРА**

1. Sarychev O. P. Optimal regressors search subjected to vector autoregression of unevenly spaced TLE series / O. P. Sarychev, B. A. Perviy, // Системні технології. – Випуск 2 (121). – 2019. – С. 95–110.
2. Sarychev O. P. Autoregression models of space objects movement represented by TLE elements / O. P. Sarychev, B. A. Perviy // Системні технології. – Випуск 2 (127). – 2020. – С. 103–116.
3. Сарычев А. П. Идентификация параметров систем авторегрессионных уравнений при известных ковариационных матрицах / А. П. Сарычев

// Международный научно-технический журнал “Проблемы управления и информатики”. – 2012. – № 3. – С. 14–30.

4. Сарычев А. П. Исследование методом статистических испытаний итерационной процедуры для идентификации параметров системы авторегрессионных уравнений / А. П. Сарычев // Системні технології. – 2014. – Випуск 3 (92). – С. 77–89.

5. Сарычев А. П. Линейная авторегрессия на основе метода группового учёта аргументов в условиях квазиповторных наблюдений / А. П. Сарычев // Искусственный интеллект. – 2015. – № 3–4 (69–70). – С. 105–123.

6. Александр Сарычев. Моделирование сложных систем в условиях структурной неопределённости: регрессионные и авторегрессионные модели / А. П. Сарычев. - LAP LAMBERT Academic Publishing RU, Saarbrücken, Deutschland, 2016. – 274 с.

7. FSCC TLE Source [Электронный ресурс]. – Режим доступа <https://www.space-track.org/#Landing>.

8. Справочник по теории вероятностей и математической статистике / В. С. Корольук, Н. И. Портенко, А.В. Скороход, А. Ф. Турбин. – М. : Наука, 1985. – 640 с.

#### REFERENCES

1. Sarychev O. P. Optimal regressors search subjected to vector autoregression of unevenly spaced TLE series / O. P. Sarychev, B. A. Perviy, // Sistemni` tekhnologi`yi. – Vipusk 2 (121). – 2019. – S. 95–110.

2. Sarychev O. P. Autoregression models of space objects movement represented by TLE elements / O. P. Sarychev, B. A. Perviy // Sistemni` tekhnologi`yi. – Vipusk 2 (127). – 2020. – S. 103–116.

3. Sary`chev A. P. Identifikacziya parametrov sistem avtoregressionny`kh uravnenij pri izvestny`kh kovariaczionny`kh matriczakh / A. P. Sary`chev // Mezhdunarodny`j nauchno-tekhnicheskij zhurnal “Problemy` upravleniya i in-formatiki”. – 2012. – # 3. – S. 14–30.

4. Sary`chev A. P. Issledovanie metodom statisticheskikh ispy`tanij iteraczi-onnoj procedury` dlya identifikaczii parametrov sistemy` avtoregressionny`kh uravnenij / A. P. Sary`chev // Sistemni` tekhnologi`yi. – 2014. – Vipusk 3 (92). – S. 77–89.

5. Sary`chev A. P. Linejnaya avtoregressiya na osnove metoda gruppovogo uchyota argumentov v usloviyakh kvazipovtorny`kh nablyudenij / A. P. Sary`chev // Iskusstvenny`j intellekt. – 2015. – # 3–4 (69–70). – S. 105–123.

6. Aleksandr Sary`chev. Modelirovanie slozhny`kh sistem v usloviyakh strukturalnoj neopredelyonnosti: regressionny`e i avtoregressionny`e modeli / A. P. Sary`chev. LAP LAMBERT Academic Publishing RU, Saarbrücken, Deutschland, 2016. – 274 s.

7. FSCC TLE Source [Е`lektronny`j resurs]. – Rezhim dostupa <https://www.space-track.org/#Landing>.

8. Spravochnik po teorii veroyatnostej i matematicheskoy statistike / V. S. Korolyuk, N. I. Portenko, A.V. Skorokhod, A. F. Turbin. – M. : Nauka, 1985. – 640 s.

Received 28.11.2020.

Accepted 01.12.2020.

### **Авторегресійні моделі руху великих фрагментів космічного**

*Розроблено статистичний метод моделювання руху великих фрагментів космічного сміття в класі авторегресійних моделей. Метод дозволяє підвищити якість опису та прогнозування руху великих фрагментів космічного сміття на основі часових рядів їх TLE-елементів.*

*На основі результатів, отриманих в процесі моделювання двох груп великих космічних об'єктів (космічних апаратів і відпрацьованих ступенів ракет-носіїв), розроблена система моделювання, яка включає в себе: визначення оптимального обсягу навчальних вибірок при моделюванні часових рядів TLE-елементів; визначення порядку авторегресії для кожного елемента-змінної; визначення оптимальної структури та ідентифікація параметрів моделі авторегресії для кожного елемента-змінної в умовах неравноотдалєнних спостережень; встановлення особливостей поведінки середньоквадратичних помилок авторегресійних моделей в часі.*

### **Autoregression models of large space debris motion**

*A statistical method was developed for modeling the large space debris motion in the class of autoregressive models. The method improves the quality of description and forecasting of the movement of large fragments of space debris based on their TLE elements.*

**Саричев Олександр Павлович** – доктор технічних наук, провідний науковий співробітник, Інститут технічної механіки НАН України і ДКА України.

**Первій Богдан Андрійович** – молодший науковий співробітник, Інститут технічної механіки НАН України і ДКА України.

**Сарычев Александр Павлович** - доктор технических наук, ведущий на-учно сотрудник, Институт технической механики НАН Украины и ГКА Украины.

**Первый Богдан Андреевич** - младший научный сотрудник, Ин-ститут технической механики НАН Украины и ГКА Украины.

**Sarichev Oleksandr Pavlovich** - Doctor of Technical Science, Principal Researcher, Institute of Technical Mechanics of the National Academy of Sciences of Ukraine and State Space Agency of Ukraine.

**Perviy Bogdan Andriyovich** - Junior Researcher, Institute of Technical Mechanics of the National Academy of Sciences of Ukraine and State Space Agency of Ukraine of Ukraine.