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PROBABILISTIC CHARACTERISTICS OF WIRELESS NETWORKS WITH INFRASTRUCTURE TOPOLOGY

Abstract. Based on the analysis of the operation of networks IEEE 802.11 DCF, a function is proposed for determining the probability of frame transmission to a central node depending on the number of stations operating in saturation mode. The probabilities of collisions are calculated. Using a polynomial approximations an expression is obtained for the network throughput, which explicitly depends on the number of the simultaneously operating stations.
Keywords: wireless network IEEE 802.11, DCF mode, collisions, throughput, probabilistic characteristics.

Introduction. Wireless networks with infrastructure topology of IEEE 802.11 standard are intensity developed. The data exchange speeds of several gigabits per second, which have already really been reached at the present time, make it possible to use them instead of wired networks, especially on twisted pair. This opens up the possibility of using wireless networks in the shops of industrial enterprises, at production sites. But the high level of electromagnetic interference, which is characteristic for these industrial premises, leads to a distortion of a significant number of frames moving from peripheral stations to the central node. Herewith these stations do not receive confirmations from the central node on sent frames. The collision compensation method CSMA/CA used in this case does not distinguish between the frame damage due to external interference from operating technology equipment and the collision that occurs when frames are mutually damages by two stations operating in the same wireless segment. In both cases the contention windows mechanism is activated, which delays the frames sent by the stations, and thereby significantly reduces the speed of information exchange in a wireless network.

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Fundamental analysis of IEEE 802.11 network using Distributed Coordination Function (DCF) was carried out in article [1] and later updated in [2]. This analysis is based on the use of bidimentional Markov chain model with ideal channel conditions and saturated load. Each frame needs to wait for a random backoff time before transmitting. The backoff is performed in discrete time units called slots and the stations are synchronized on the slot boundaries. As a result of this analysis, it was obtain that the probability τ that a station transmits in a randomly chosen slot time is

$$\tau = \frac{2(1-2p)}{(1-2p)(W_0+1)+pW_0[1-(2p)^m]}, \tag{1}$$

where $W_0=CW_{min}$ is minimum contention window, $CW_{max}= 2^m W_0$.

The probability p that a transmitted frame encounters a collision is defined as the probability that, in a time slot, at least one of the $n-1$ remaining stations transmit

$$p = 1-(1-\tau)^{n-1} \tag{2}$$

As can be seen from expressions (1) and (2) the dependence of the collision probability p on the number of simultaneously operating stations n at a given initial window width W_0 can be obtained only by rather cumbersome numerical calculations. At the same time it is useful for engineering calculations to have analytical expressions for explicitly calculations the probability of collisions and the network throughput based on this, depending on the number of stations operating in saturation mode. The proposed article is devoted to solving this problem.

Problem statement. The purpose of this article is to determine the dependence of throughput of an infrastructure of wireless network IEEE 802.11 operating in DCF mode on the number of workstations in an explicit analytical form suitable for engineering calculations.

Main part. If p is the collision probability, then an arbitrary frame is success-fully transmitted with probability $(1-p)$, and the average backoff window is $(W_0-1)/2$. If the first transmission fails, the frame is successfully transmitted on the second attempt with probability $p(1-p)$. The average backoff window W_{avg} in this case is $(2W_0-1)/2$. On the third attempt to transfer $W_{avg}= (4W_0-1)/2$ and so on. It is logical to assume [3] that the probability of successful frame transfer τ back is proportional to the average window width, i.e. $\tau = 1/W_{avg}$.

For the probability of frame transmission to the central node of the segment, we propose the following expression

$$\tau = \frac{2}{2^{(\frac{n}{N})^q} \cdot W_0 - 1}, \tag{3}$$

where n is the number of stations operating in saturation mode, N is the maximum number of stations, W_0 is the initial window width, q is the variable parameter.

The collision probability, defined by expression (2), for fixed values N and W0, depends only on the number of operating stations.

When operating a wireless network in a production environment with a high level of electromagnetic interference due to operating equipment, parameter BER (Bit Error Rate) [4] can reach BER = $10^{-3}...10^{-4}$. Under these conditions, it is advisable to limit N to 20 (N = 20).

The graphs of the dependences of $\tau(n)$ and $p(n)$ for different values of q are presented in Figs. 1 and 2, respectively.

The dependencies shown in the Fig.1 and Fig.2 are well approximated by parabolas:

$$\begin{aligned}
&\text{for } q=2 \quad \tau(n)=10^{-4}\,(-0{,}5n^2 - 7n + 659),\ R^2 = 0{,}9984;\\
&\text{for } q=3 \quad \tau(n)=10^{-4}\,(-n^2 + 6n + 636),\ R^2 = 0{,}9961;\\
&\text{for } q=4 \quad \tau(n)=10^{-4}\,(-n^2 + 15n + 616),\ R^2 = 0{,}9951;
\end{aligned} \tag{4}$$

$$\begin{aligned}
&\text{for } q=2 \quad p(n)=10^{-3}\,(-2{,}2n^2 + 67{,}2n - 60{,}2),\ R^2 = 0{,}9996;\\
&\text{for } q=3 \quad p(n)=10^{-3}\,(-2{,}3n^2 + 71{,}9n - 70{,}4),\ R^2 = 1{,}0;\\
&\text{for } q=4 \quad p(n)=10^{-3}\,(-2{,}2n^2 + 73{,}1n - 73{,}9),\ R^2 = 0{,}9998.
\end{aligned} \tag{5}$$

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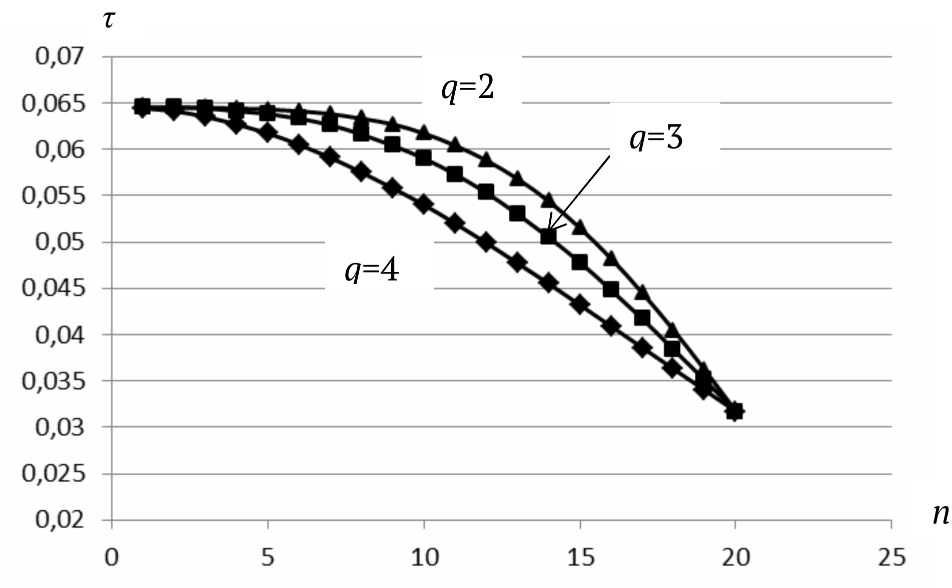

Figure1 - Dependencies of the probability of frame transmission τ on the number n of working stations

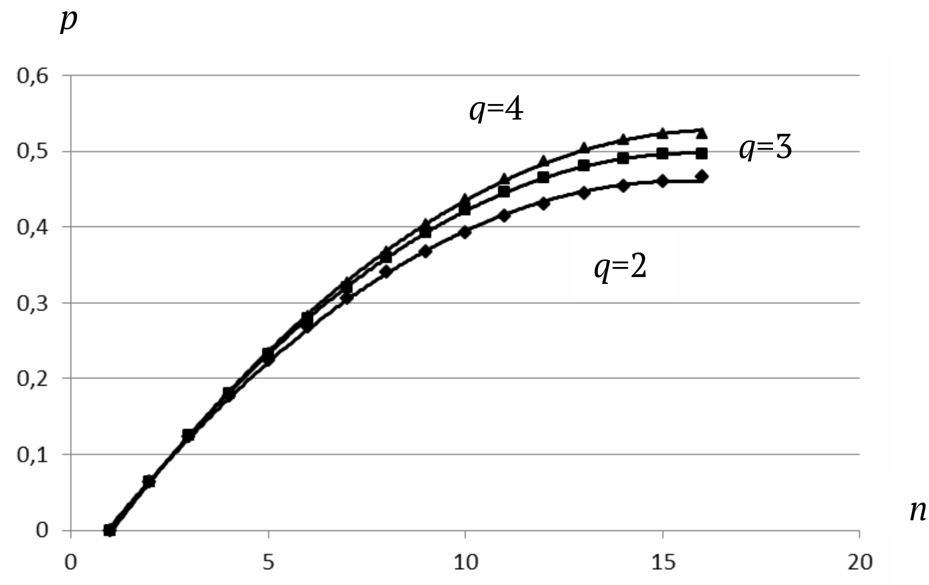

Figure 2 - Dependencies of the probability of frame collision p on the number n of working stations

It should be noted that the approximation of the functions $p(n)$ is carried out in the range $n = 1...16$. Further, with an increase in n to the limiting value $n = N$, a decrease in the functions $p(n)$ is observed for $q = 3$ and $q = 4$. Such decrease for $q = 2$ is observed after $n = 18$.

In accordance with [1] the network throughput S is defined as ratio of the average frame payload of information transmitted in a time interval to the average length of the interval in form

$$S = \frac{P_{tr} \cdot P_s \cdot E[Fr]}{(1-P_{tr})\eta\sigma + P_{tr}P_sT_s + P_{tr}(1-P_s)T_c}, \tag{6}$$

where $E[Fr]$ is the average frame payload; P_{tr} is the probability that there is at least one transmission in a considered time interval, i.e. $P_{tr} = 1-(1-\tau)n$; P_s is the probability that a transmission occurring in the channel is successful, i.e. $P_s = 1 - p$ and

$$P_s = \frac{n\tau(1-\tau)^{n-1}}{1-(1-\tau)^n}; \tag{7}$$

η is the number of the empty slots; σ is the slot duration; T_s and T_c are the average times the channel is busy because of successful transmission and collision correspondingly.

Due to DCF

$$T_s = T_{phy} + \frac{E[Fr]}{R} + SIFS + \delta + ACK + DIFS + \delta \tag{8}$$

$$T_c = T_{phy} + \frac{E[Fr]}{R} + DIFS + \delta, \tag{9}$$

where T_{phy} is the duration of physical header transmission, R is the information trans-mission rate, $SIFS$ – Short Interframe Space, $DIFS$ – DCF Interframe Space, ACK – Acknowledgment, and δ is the propagation delay.

The calculation results show that expression (6) can be represented as

$$S[Mbps] = \frac{10^{-7} n \cdot L_4(n) \cdot E[Fr]}{B_2(n) \cdot \eta\sigma + 10^{-7} nL_4(n)(T_s - T_c) + C_2(n)T_c} \tag{10}$$

where $L_4(n) = \sum_{k=0}^{4} a_{4-k}n^k$, $B_2(n) = \sum_{k=0}^{2} b_{2-k}n^k$, $C_2(n) = \sum_{k=0}^{2} c_{2-k}n^k$.

The values of the coefficients of polynomials $L_4(n)$, $B_2(n)$, and $C_2(n)$ are given in Tabl.1.

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Table 1

Polynomials coefficients

q	a_0	a_1	a_2	a_3	a_4	b_0	b_1	b_2	c_0	c_1	c_2
q = 2	-0,95	18,55	1172,4	-49337	692741	1,9	-63,7	1051	-2,2	67,2	-60,2
q = 3	-2,3	85,8	-40,2	-39366	681156	2,3	-72	1071	-2,3	71,9	-70,4
q = 4	-2,4	112,6	-753,7	-30939	667190	2,4	-76,6	1083	-2,2	73,1	-73,9

As follows from the graphs presented in Fig.1 and Fig.2, expressions (4) and (5), as well as coefficients of the approximating polynomials in Tabl.1, the value q = 3 is optimal.

For this value q, the dependence of network throughput S on the number of stations operating in the saturation mode was calculated. The dependency graph is shown in Fig.3. The calculation was carried out using the following parameter values:

T_{phy} = 48μs, $E[Fr] = 8\cdot 10^3$ bit, $\eta = 1$, R = 100 Mbps, σ = 9 μs, $SIFS$ = 10 μs, $DIFS$ = 50 μs, ACK = 1,12 μs, δ = 0,33 μs.

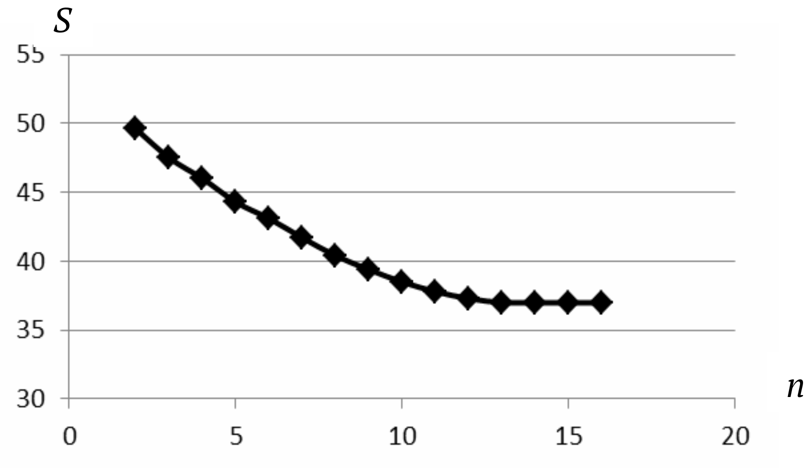

Figure 3 - Dependence of the throughput S on the number n of working stations

The nature of the presented dependence and its main parameters correspond to the throughput validation results obtained using the simulator ns-3 and presented in [7]

Conclusions. Based on the analysis of the operation of networks IEEE 802.11 DCF, a function is proposed for determining the probability of frame transmission to the central node of a wireless network IEEE 802.11 DCF, depending on the number of peripheral stations in the network operating in saturation mode. Collision probabilities calculated.

Using approximated polynomials of the second and fourth degrees, an expression is obtained for the network throughput in an explicit form depending on the number of simultaneously operating stations. The nature of this dependence and its main parameters correspond to the results presented in literature and obtained using the simulator ns-3.

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Імовірнісні характеристики роботи бездротових мереж з інфраструктурною топологією

Базуючись на аналізі функціонування мереж IEEE 802.11 DCF, запропонована функція для визначення імовірності передачі фрейма центральному вузлу в залежності від кількості станцій, працюючих в режимі насичення. Розраховані імовірності колізій. З використанням поліноміальної апроксимації одержано вираз для смуги пропускання мережі, яка у явному вигляді залежить від кількості одночасно працюючих станцій.

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Вероятностные характеристики работы беспроводных сетей с инфраструктурной топологией

Базируясь на анализе функционирования сетей IEEE 802.11 DCF, предложена функция для определения вероятности передачи фрейма центральному узлу в зависимости от количества станций, работающих в режиме насыщения. Рассчитаны вероятности коллизий. С использованием полиномиальной аппроксимации получено выражение для полосы пропускання сети, которая в явном виде зависит от количества одновременно работающих станций.

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