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L.G. Akhmetshina, A.A. Yegorov

IMPROVING THE SENSITIVITY AND RELIABILITY OF GRAYSCALE LOW-CONTRAST IMAGES SEGMENTATION BASED ON THE ITERATIVE APPLICATION OF TYPE-2 FUZZY TRANSFORMATIONS

Annotation. When processing images, one of the most difficult tasks is their segmentation, which is due to the lack of a priori information about the presence, objects of interest shape and location, the presence of low-contrast areas in which the sought-after objects of interest may be partially or completely located, noise, image blur and other factors. Currently, one of the common approaches to solving this problem is various algorithms based on the applying of type-1 and type-2 fuzzy sets usage. This paper proposes to improve the sensitivity and reliability of the iterative algorithm for segmentation of grayscale images by changing the membership functions calculating method, as well as reducing the number of control parameters. Experimental results are presented using the example of segmentation of real grayscale medical images.
Keywords: low-contrast images, fuzzy methods, visual analysis, membership function, segmentation, fuzzy sets of type-1, fuzzy sets of type-2.

Introduction. Currently, the fuzzy sets usage allows us to significantly increase the efficiency of algorithms in solving various problems related to image processing [1, 2]. In particular, when performing the image segmentation task, which is traditionally considered the most complex and ambiguous operation, fuzzy logic allows one to take into account both the uncertainty factors associated with the system of their formation (measurement error, noise, resolution), and with the inaccuracy of information about the objects of interest, where the boundaries of the objects are blurred or noisy (for example, in medicine).

Statement of the problem. The purpose of this article is to demonstrate the possibility of improving the sensitivity and reliability of low-contrast grayscale images segmentation in a method with iterative application of fuzzy transformations by changing the way for calculating type-1 and type-2 membership functions, as well as reducing the number of control parameters.

Analysis. For images, the transition to a fuzzy space (fuzzification) is carried out on the basis of the original brightness values transformation into membership functions of fuzzy sets corresponding to the analyzed properties, for example, the gray level.

One of the possible ways to perform such a transformation is to use the fuzzy clustering algorithm FCM (Fuzzy C-Means) proposed by Bezdek in 1981 [4]. For each pixel, it allows us to obtain a set of membership functions, the number of which corresponds to the number of

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clusters with different properties. For grayscale images, membership functions in numerical form (values from 0 to 1) represent the degree of uncertainty in the original brightness values of each pixel. The new multidimensional fuzzy feature space can be used to solve various image analysis problems [5]. In particular, segmentation is a visualization of the clustering results.

The application of type-2 fuzzy sets introduced by Zadeh [6] and algorithms using them (e.g. T2FCM presented by Rhee and Hwang [5]) extends the capabilities of type-1 fuzzy sets. Type-2 fuzzy sets reflect the concept of "fuzzy degree of membership" and provide the ability to describe fuzziness in a fuzzy set. Type-2 membership functions (MFT2) are generated based on type-1 fuzzy membership functions (MFT1) and are capable of modeling uncertainty in the solution, due to which the initial data are grouped more correctly and accurately [7, 8].

The disadvantage of FCM algorithms and most of its modifications based on the type-2 fuzzy sets usage is the need for an a priori the number of clusters assignment and the ambiguity of the defuzzification stage.

The number of clusters choice has a significant impact on the segmentation result, since it determines the level of detail of the result and, if the value is excessive, can lead to the formation of artifacts. Determining the optimal number of clusters value is a challenging task associated with the study of a priori data for each specific analyzed subject area and the problem being solved (for example, the number of components in materials science, tissues in medicine, objects in flaw detection).

Existing modifications of the FCM algorithm with automatic determination of the optimal number of clusters (for example, described in [9]) are characterized by lower operating speed and higher computational complexity. In addition, such methods often remain dependent on the correctness of the initial number of clusters, which reduces the degree of automation of the process.

When solving segmentation problems, this complexity can be minimized by moving from pixel-by-pixel analysis to analysis of image histogram characteristics. Histogram and local contrast methods are used as an alternative approach to reduce the load when moving into fuzzy space [10]. Unlike clustering, the calculation of the membership function here is based on a direct assessment of pixel brightness without using an iterative search for centroids. Although such methods are applicable to segmentation problems, the lack of adaptive modification of the membership function limits the ability to control the detail of the final result.

In the works [11, 12], an iterative algorithm for segmentation of grayscale low-contrast images was proposed based on type-1 and type-2fuzzy sets usage, which doesn't require an a priori assignment of the number of fuzzy clusters and the calculation of the centroid matrix.

The main part. In this paper, a modification of the iterative segmentation algorithm [11, 12] is proposed, operating MFT1 and MFT2 without using the centroid matrix. The criteria for stopping the process are the specified training error (reflecting the change in MFT2 in adjacent iterations) and the number of cycles limitation. Changing the method for calculating MFT1 and MFT2 allowed us to reduce the number of control parameters, which simplified the algorithm adaptation to the brightness characteristics of the analyzed images.

In the presented algorithm, MFT1 is initially calculated based on the input data, and MFT2 is calculated as the difference between the “upper” u_h and “lower” u_l MFT1 (Fig. 1), which allows for a complete description of the MFT1 blur (FOU – the footprint of uncertainty). The values of u_h and u_l MFT1 are calculated based on the average of two different power transformations.

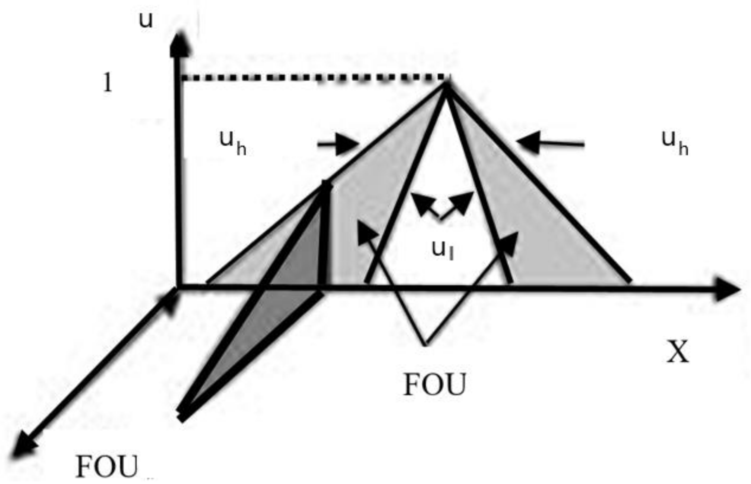

Figure 1 – Fuzzy sets of type-2: the boundaries of uncertainty

The algorithm of the modified method proposed in this paper contains the following steps:

1. Scaling the input grayscale image to the range $[0,1]$.

2. Calculation of the matrix u^0 MFT1 (the dimension $x_{\max} \times y_{\max}$ coincides with the input image) initial value based on the scaled image I obtained in the previous step:

$$u^0_{x,y} = \left| \left(I_{x,y}\right)^{1-\operatorname{sgn}\left(I_{x,y}-0.5\right)} \cdot \left(P_{x,y}\right)^{1+\operatorname{sgn}\left(I_{x,y}-\bar{I}\right) \cdot P_{x,y}} \right|, \tag{1}$$

$$P_{x,y} = \left(I_{x,y}\right) / \left(I_{x,y} + c_1\right), \tag{2}$$

$$C_1 = 0.5 + 1.25 \cdot C_u - \left(1 - C_u\right) \cdot \bar{I}, \tag{3}$$

$$C_u = \frac{\bar{I}}{\left(1-\bar{I}\right) \cdot m_c}, \tag{4}$$

$$m_c = \left(\left(\frac{1}{\overline{I}_2} \right)^{-\overline{I}_1} \right)^{0.5+1.2\cdot\overline{I}_2}, \tag{5}$$

$$\overline{I}_2 = \left(\overline{I}_1\right)^{1+0.1\cdot\overline{I}_1}, \tag{6}$$

$$\overline{I}_1 = \frac{\max\left(\bar{I}, 0.5-\bar{I}\right)+0.5}{2}, \tag{7}$$

where $\bar{I}$ is average on image I.

3. Calculation of the current matrix MFT2 (a^t), the dimension of which also coincides

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with I, based on the difference between the u_h^t and u_l^t matrices of the current iteration as follows:

$$\left(u_{h1}^{t}\right)_{x,y} = \left(\left(u^{t}\right)_{x,y}\right)^{1-\left(\left(u^{t}\right)_{x,y}\right)^{1-\operatorname{sgn}\left(\left(u_{1}^{t}\right)_{x,y}-0.5\right)\cdot\left(\left(u^{t}\right)_{x,y}\right)^{1-\left(u^{t}\right)_{x,y}}}} - \frac{C_f}{2.5-\left(u_{1}^{t}\right)_{x,y}}, \tag{8}$$

$$\left(u_{l1}^{t}\right)_{x,y} = \left(\left(u^{t}\right)_{x,y}\right)^{1+\left(\left(u^{t}\right)_{x,y}\right)^{1+\operatorname{sgn}\left(\left(u_{1}^{t}\right)_{x,y}-0.5\right)\cdot\left(\left(u^{t}\right)_{x,y}\right)^{1+\left(u^{t}\right)_{x,y}}}} + \frac{C_f}{1.5+\left(u_{1}^{t}\right)_{x,y}}, \tag{9}$$

$$\left(u_{h2}^{t}\right)_{x,y} = \left(\left(u^{t}\right)_{x,y}\right)^{C_f+0.25+\left(\overline{\left(\cdot u^{t}\right)}^{1}+0.5-C_f\right)/\left(2+C_f\right)}, \tag{10}$$

$$\left(u_{l2}^{t}\right)_{x,y} = \left(\left(u^{t}\right)_{x,y}\right)^{C_f+0.5+\overline{\left(\cdot u^{t}\right)}^{1}+\left(\overline{\left(\cdot u^{t}\right)}^{1}+0.5+C_f\right)/\left(2-C_f\right)}, \tag{11}$$

$$\left(u_{1}^{t}\right)_{x,y} = \frac{u_{x,y}^{t}+\overline{\left(u^{t}\right)}^{1}}{2}, \tag{12}$$

$$\overline{\left(u^{t}\right)}^{1} = \left(\overline{u^{t}}\right)^{1-\max\left(\overline{u^{t}},1-\overline{u^{t}}\right)}, \tag{13}$$

$$C_u^{'} = \frac{\overline{u^{t}}}{C_u+\left(0.5+u^{t}\right)/2}, \tag{14}$$

$$C_f = \left(C_u^{'}\right)^{1+C_u^{'}/\left(1.5+\overline{u^{t}}\right)}, \tag{15}$$

$$\left(u_{h}^{t}\right)_{x,y} = \left(\left(u_{h1}^{t}\right)_{x,y}+\left(u_{h2}^{t}\right)_{x,y}\right)/2, \tag{16}$$

$$\left(u_{l}^{t}\right)_{x,y} = \left(\left(u_{l1}^{t}\right)_{x,y}+\left(u_{l2}^{t}\right)_{x,y}\right)/2, \tag{17}$$

where $\overline{u^t}$ is the average on current matrix MFT1 u^t (at the 1st iteration, the matrix u^0 is used, and on subsequent ones matrix u^{t-1} (obtained at the end of the previous iteration) is used).

4. Starting from the 2nd iteration of training:

4.1. The value Δ^t is calculated by the formula:

$$\Delta^{t} = \sum_{x=1}^{x_{\max}} \sum_{y=1}^{y_{\max}} \left| a_{x,y}^{t} - a_{x,y}^{t-1} \right|, \tag{18}$$

where a^t and a^{t-1} are MFT2 matrices current and previous iterations, respectively.

4.2. If the next condition is met:

$$\Delta^{t} \geq \Delta^{t-1}, \tag{19}$$

where Δ^{t-1} is the value, obtained by the formula (18) at the previous iteration (initially initialized with a very large number, unattainable in practice), then instead of the current values of the matrices a^t, u_l^t and u_h^t the values a^{t-1}, u_l^{t-1} and u_h^{t-1}, respectively, obtained at the end of the previous iteration, are written and the learning process stops (go to step 7). This step is necessary to prevent the overfitting effect.

4.3. If the next condition is true:

$$\Delta^t < \varepsilon , \tag{20}$$

where ε is the specified training accuracy, the training process stops (go to step 7).

5. If the next condition is met:

$$t < t_{\max} , \tag{21}$$

where $t_{\max}$ is maximal number of iterations, then the matrix u^{t+1} is calculated (by the formulas (1) – (7), in this case u^t and $\overline{u^t}$ are used instead of I and $\bar{I}$, accordingly), which is required for calculating the matrix a^t at the next iteration.

6. When condition (21) is met, the transition to the next training iteration occurs (step 3).

7. Matrix a^t, u_l^t and u_h^t values are scaled to the range $[0,1]$.

8. The matrix I^w is calculated by the formulas:

$$I_{x,y}^w = \left(u_h^t\right)_{x,y}^{1-\frac{a_{x,y}^t}{a_{x,y}^t + d_1}} - \left(u_l^t\right)_{x,y}^{1-\frac{a_{x,y}^t}{a_{x,y}^t + d_1}} , \tag{22}$$

$$d_1 = 2 + \overline{u_l^t} , \tag{23}$$

where $\overline{u_l^t}$ is the average on the matrix u_l^t, after that I^w is scaled to the range $[0,1]$ and interpreted as a grayscale image.

9. The output image I^{out} is formed based on the weighted sum as follows:

$$I_{x,y}^{out} = I_{x,y}^h \cdot C_{out} + I_{x,y}^a \cdot \left(1 - C_{out}\right) , \tag{24}$$

$$C_{out} = \left(\overline{I^w} + \overline{u_l^t}\right) / 2 , \tag{25}$$

where I^h and I^a are grayscale images resulting from the application of equalization and adaptive equalization of histogram methods (with a uniform transformation function) to the image I^w, accordingly, and $\overline{I^w}$ is the average on the image I^w.

The experimental results were obtained on the example of processing various medical images. Examples of such images are shown in Fig. 2a, 3a (Fig. 2b and 3b show their histograms). Figure 2a shows a tomogram of the brain taken for the purpose of diagnosing the presence of a hematoma and determining the area of its influence if detected (the area of interest is indicated by a rectangle). Fig. 3a shows an X-ray of the cervical spine.

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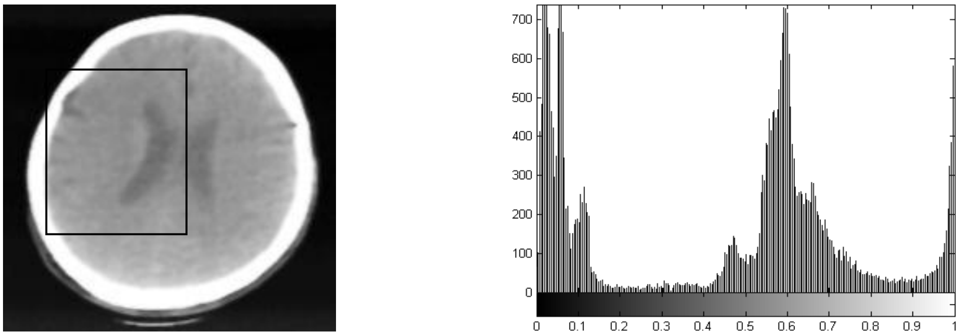


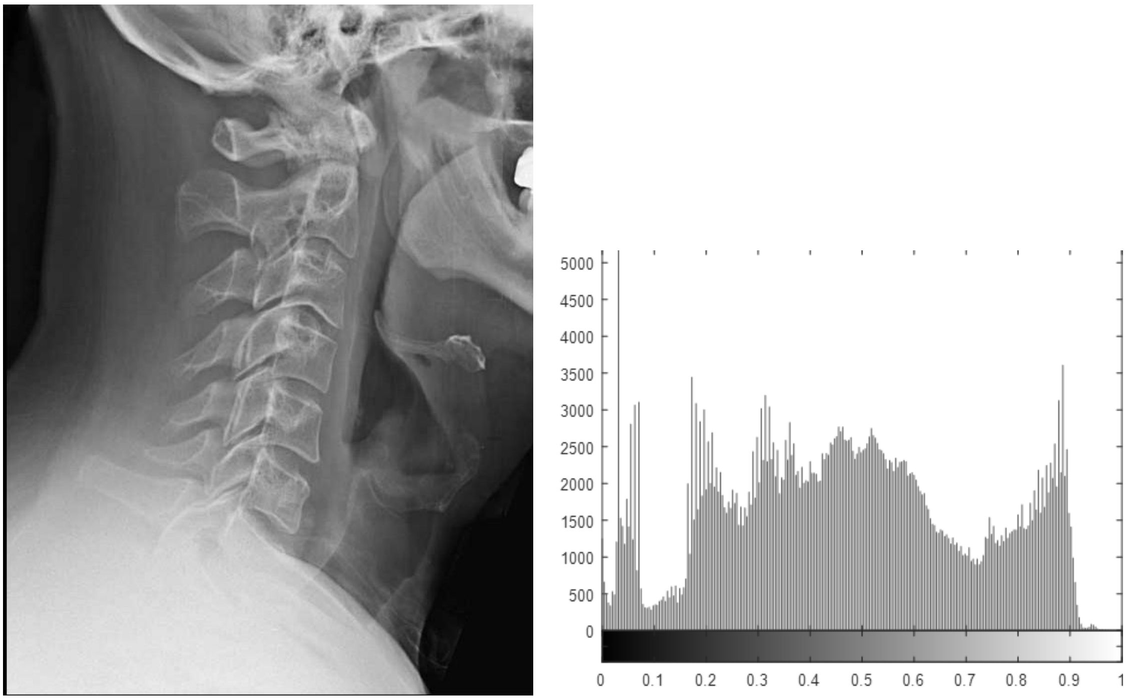
a b

Figure 2 – X-ray tomography of the brain: a – original grayscale image (204x201); b – its histogram

When conducting experiments using the proposed algorithm, the following control parameters were used: $\varepsilon = 0.035$, the maximal number of iterations was 12 (the number of training iterations practically did not exceed 4-5). Original image visualization after scaling it to the range $[0,1]$ was also carried out on the basis of formula (24), where $C_{out} = \bar{I}$.

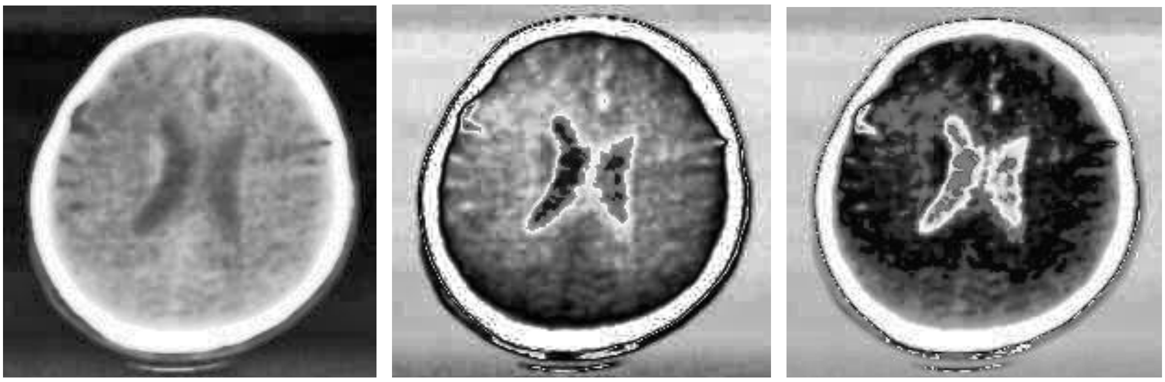
Segmentation of the brain tomogram shown in Fig. 2a shows that the modified method (Fig. 4c) allows for a more precise identification of the hematoma's area of influence, as well as the overall brain structure, compared to the original method (Fig. 4b). Original image visualization based on formula (24) does not allow for a clear identification of the hematoma's area of influence (Fig. 4a).

Segmentation of the X-ray image shown in Fig. 3a using the proposed modified method also allows for a clearer identification of the cervical spine structure and parts of the skull (Fig. 5b) compared to the original method usage (Fig. 5a).



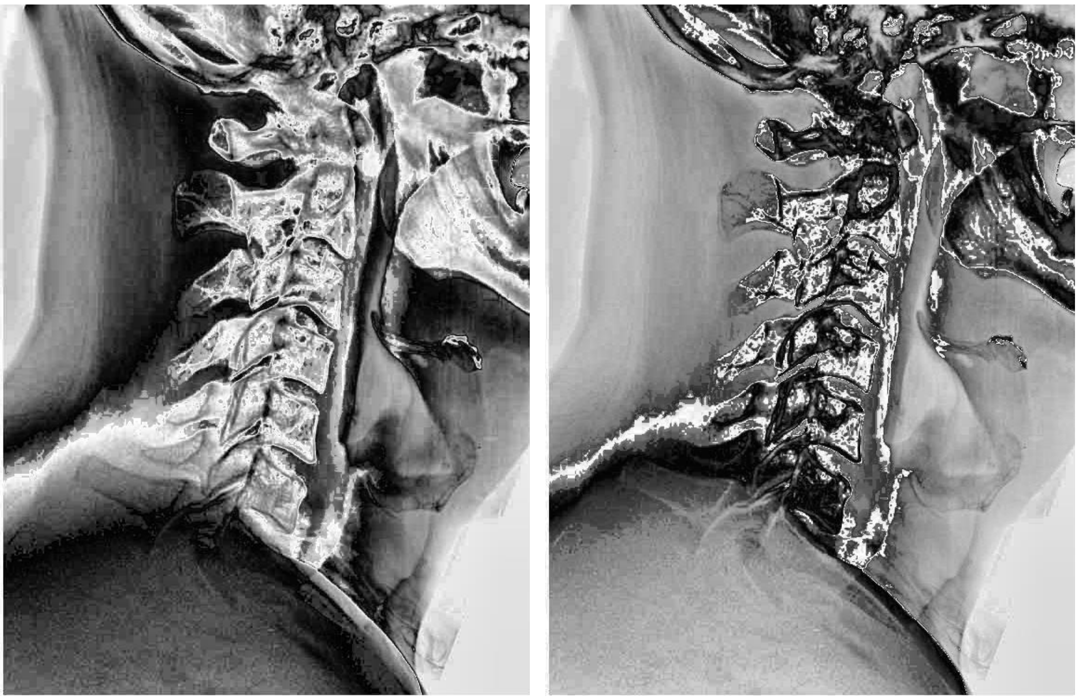
a b

Figure 3 – X-ray image: a – original grayscale image (744x570); b – its histogram



a b c

Figure 4 – Segmentation of the brain tomogram (Fig. 2a): a – original image visualization according to formula (24); b – by original and c – by modified methods



a b

Figure 5 – Segmentation of X-ray image (Fig. 3a):
a – by original and b – by modified methods

Conclusions:

– the usage of the proposed modification allows for improved segmentation detail, providing a clearer identification of the areas of interest boundaries and the image internal structure in comparison with the basic algorithm;

– the modified method is characterized by a smaller number of control parameters, which simplifies its configuration and adaptation to specific tasks;

– a promising direction for further research is the development of new approaches to the formation of MFT1 and MFT2 to improve the algorithm's resistance to noise.

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Підвищення чутливості та достовірності сегментації напівтонових слабкоконтрастних зображень на основі ітеративного застосування нечітких перетворень типу-2

Сегментація є одним із найскладніших завдань, що вирішуються при обробці зображень, що обумовлено відсутністю апріорної інформації про наявність, форму і розташування об'єктів інтересу, наявністю низькоконтрастних областей, в яких об'єкти інтересу, що шукаються, можуть частково або повністю розташовуватися, та іншими факторами.

В даний час одним з поширених підходів до вирішення цієї задачі є застосування різних алгоритмів, заснованих на використанні нечітких множин типу-1, таких як FCM та його модифікації, і типу-2, наприклад, T2FCM. Однак ці методи є алгоритмами кластеризації, тому вимагають розрахунку матриці центроїдів, а також завдання кількості нечітких кластерів, хоча для виконання безпосередньо сегментації це необов'язково. Існують також методи підвищення якості зображень, засновані на застосу-

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ванні нечітких множин, які можна використовувати для кластеризації зображень, однак вони не є ітеративними, що не дозволяє керувати деталізацією при сегментації.

У роботі запропоновано підвищення чутливості та достовірності ітеративного алгоритму сегментації напівтонових зображень за рахунок зміни способу розрахунку функцій належності, а також зниження числа керуючих параметрів.

Запропонований алгоритм сегментації не використовує матрицю центроїдів, не вимагає завдання числа нечітких кластерів, є ітеративним, має мале число керуючих параметрів (точність навчання та максимальне число ітерацій). Описаний у роботі метод заснований на застосуванні нечітких функцій приналежності типу-1 (НФП1) і типу-2 (НФП 2), при цьому розрахунок матриці НФП2 заснований на різниці матриць "верхньої" та "нижньої" НФП1, які, у свою чергу, розраховуються як середнє за двома типами степеневих перетворень. Формування результуючого зображення здійснюється на основі зваженої суми результатів застосування методів еквалізації та адаптивної еквалізації гістограми до напівтонового зображення, сформованого на основі значень матриць НФП2 і "верхньої" і "нижньої" НФП1 після завершення навчання.

Наведені в роботі експериментальні результати на прикладі сегментації реальних напівтонових медичних зображень показують підвищення деталізації при використанні запропонованої модифікації, а також більш чітке виділення меж областей інтересу і внутрішньої структури зображення при меншій кількості керуючих параметрів у порівнянні з базовим алгоритмом. Перспективним напрямом подальших досліджень є розробка нових підходів до формування НФП1 та НФП2 для підвищення стійкості алгоритму до шумів.

Ключові слова: слабкоконтрастні зображення, нечіткі методи, візуальний аналіз, функція приналежності, сегментація, нечіткі множини типу-1, нечіткі множини типу-2.

Ахметшина Людмила Георгіївна – доктор технічних наук, професор, професор кафедри електронних обчислювальних машин, Дніпровський національний університет ім. О. Гончара.
ORCID: https://orcid.org/0000-0002-5802-0907
Єгоров Артем Олександрович − старший викладач кафедри комп'ютерних наук та інформаційних технологій, Дніпровський національний університет ім. О. Гончара.
ORCID: https://orcid.org/0000-0002-7558-785X

Akhmetshina Liudmyla Georgievna − Doctor of Technical Sciences, professor, professor of the department of electronic computers, Oles Honchar Dnipro National University.
ORCID: https://orcid.org/0000-0002-5802-0907
Yegorov Artyom Alexandrovich − Senior Lecturer of Computer Science and Information Technologies Department, Oles Honchar Dnipro National University.
ORCID: https://orcid.org/0000-0002-7558-785X