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HYBRID MODELING OF EEG: THE FITZHUGH-NAGUMO-LORENZ MODEL

Abstract. The paper presents a method of modeling electroencephalographic (EEG) signals using a hybrid biophysical model that combines the FitzHugh-Nagumo dynamics and the chaotic Lorentz system. A comprehensive approach to optimizing the model parameters based on neural networks is developed, which automatically adjusts the parameters to maximize the fit to real EEG data. The proposed model demonstrates the ability to reproduce the characteristic features of EEG signals, including the main rhythms and the corresponding spectral characteristics. An interactive software tool developed in MATLAB provides a convenient interface for use. The results demonstrate the potential of this approach for both neuroscientific research and clinical applications in the diagnosis and modeling of pathologies.
Keywords: EEG, FHN, FHNL, Lorenz system, neural network, modeling, parameter optimization, MATLAB.

Problem definition. The study of the principles of human brain functioning remains one of the most difficult scientific challenges in history. Significant progress has been made thanks to comprehensive research at the intersection of biology, neurochemistry, genetics, and more. However, with each new discovery of the functions and areas of brain activity, more and more additional questions arise, which is why there is still no generally accepted comprehensive theory of brain function. Therefore, one of the key tasks of the analysis and adaptability of existing research methods is to create a dynamic system that will cover the dependence of those activity parameters that most characteristically affect the studied condition factor, disease symptom, or physiological process.

Research goal. Develop a mathematical model of EEG and a controlled parameter space with manual or neural network control of brain dynamics.

Overview. Electroencephalography (EEG) is widely used in both basic neuroscience research and clinical practice. Despite significant advances in signal recording accuracy, the full potential of EEG remains unrealized. A promising area of development is the introduction of computational models for the comprehensive integration of electrophysiological data, network models (at the level of neuroimaging), and behavioral aspects.

The main objective of this work is to create a computational model that integrates cellular processes at different spatial levels and establishes clear correlations with empirical EEG data.

A model is represented as a set of equations describing the relationships between variables. It is noteworthy that there are models for various spatial scales [1, 2, 3], ranging from single-cell impulse activity to macroscopic oscillations. Equations are used to model the changes in each variable over time or, in some cases, to find analytical solutions to the interdependencies between variables. The dynamics of the resulting time series is influenced by a set of parameters that can be calculated from available data (for example, a model that simulates the excitation of a certain type of neuron may include a time constant determined from rodent experiments) or systematically modified for research purposes. The main task is to create time series that can be compared with empirical data.

The clinical application of EEG patterns is of great importance. Psychiatric and neurological disorders affect an increasingly large proportion of the population, including both patients and their caregivers, causing enormous economic and humanitarian costs to healthcare systems worldwide [4]. One of the main obstacles to improving patient care is the lack of individualized diagnosis, prognosis, and treatment planning [5]. Computational models can be adapted to individual characteristics by adjusting parameters to fit specific data (by directly setting measured parameters or by searching for values that match time series to recorded data). The optimized parameters can be linked to clinical markers, symptoms, and behaviors, which allows, for example, to differentiate pathologies. The use of personalized models provides additional diagnostic capabilities in the form of modeling parameters and results, significantly assisting clinicians in diagnosis and treatment planning [6, 7].

FitzHugh-Nagumo model (FHN). The FitzHugh-Nagumo (FHN) model was developed in the early 1960s as a simplified version of the more complex Hodgkin-Huxley model. In 1961, Richard FitzHugh proposed a mathematical simplification that preserved the key qualitative properties of neuronal dynamics but significantly reduced the computational complexity. Independently of him, Jinichi Nagumo and his colleagues created an electronic circuit in 1962 that realized these equations.

The FitzHugh-Nagumo model describes the dynamics of the neuronal membrane potential and the recovery variable through a system of two differential equations:

$$\begin{cases} \frac{dV}{dt} = V - \frac{V^3}{3} - W + I(t) \\ \frac{dW}{dt} = \epsilon(V + a - bW) \end{cases} \quad (1)$$

where V represents the membrane potential, W - recovery variable, $I(t)$ - external current (input signal), ϵ, a, b - model parameters. The parameter ϵ reflects the difference in the time scales of the potential dynamics and the recovery variable, while a and b affect the bifurcation properties of the system.

The model demonstrates important biophysical properties, including threshold excitation, all-or-nothing action potential generation, refractory period, and various dynamic modes arising from bifurcations in the parameters.

Lorentz system. The Lorentz system, developed by meteorologist Edward Lorentz in 1963 while studying atmospheric convection, is a classical three-dimensional system of non-

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linear differential equations that has become a fundamental contribution to the development of chaos theory and nonlinear dynamics.

The Lorentz system is described by the following equations:

$$\begin{cases} \frac{dX}{dt} = \sigma(Y - X) \\ \frac{dY}{dt} = X(\rho - Z) - Y \\ \frac{dZ}{dt} = XY - \beta Z \end{cases} \tag{2}$$

where X, Y та Z — state variables, σ, ρ and β - system parameters. This system demonstrates chaotic behavior at certain parameter values, in particular, the classical values $\sigma = 10$, $\rho = 28$ and $\beta = \frac{8}{3}$ lead to the formation of the Lorentz attractor.

Lorentz's key discovery was the demonstration of deterministic chaos, a phenomenon in which minimal differences in initial conditions lead to radically different trajectories over time, which revolutionized the understanding of the predictability of dynamical systems and established fundamental limitations to long-term forecasts.

Hybrid model FHNL. We integrate the FHN model and the Lorentz system into a single hybrid model. For a system with cap N neurons, the model is described by a system of $2N + 3$ differential equations:

$$\begin{cases} \frac{dV_i}{dt} = V_i - \frac{V_i^3}{3} - W_i + I_{EEG}(t) + \sum_{j=1}^{N} g_{syn} W_{ij} V_j,\ i = \overline{(1, N)} \\ \frac{dW_i}{dt} = \epsilon(V_i + a - bW_i),\ i = \overline{(1, N)} \\ \frac{dX}{dt} = \sigma(Y - X) \\ \frac{dY}{dt} = X(\rho - Z) - Y \\ \frac{dZ}{dt} = XY - \beta Z \end{cases} \tag{3}$$

where $I_{EEG}(t)$ - interpolated real EEG signal or generated input noise, g_{syn} - strength of synaptic connection, W_{ij} — matrix of connections between neurons.

Signal generation and optimization methods. The synthetic EEG signal is generated as a weighted sum of neuronal membrane potentials with the addition of high-frequency components:

$$EEG(t) = \sum_{i=1}^{N} \alpha_i V_i(t) + \eta(t) \tag{4}$$

where α_i - weighting coefficients for each neuron, $\eta(t)$ - high-frequency noise that provides realistic signal characteristics. Additionally, bandpass filtering is applied in the range of 0.5-40 Hz, which corresponds to standard EEG frequencies.

For further effective neural network training, a set of time and frequency characteristics is extracted from the EEG signal: mean value, standard deviation, skewness, kurtosis, maximum and minimum values, median, median absolute deviation, variance, power in key frequency bands, relative power, power ratio, and entropy indicators.

To predict the optimal model parameters, a feed-forward neural network with the following architecture is used:

- Input layer: the dimension corresponds to the number of extracted features;
- Two hidden layers with 20 and 10 neurons, respectively;
- Output layer: 7 neurons corresponding to the model parameters;
- Activation function: hyperbolic tangent;
- Learning algorithm: Levenberg-Marquardt.
- Inputs are normalized before training to ensure better performance.

Software implementation. The software implementation of the FHNL hybrid model, developed in MATLAB, is a comprehensive toolkit that integrates the numerical solution of a system of differential equations using the fourth- or fifth-order Runge-Kutta method (ODE45) with machine learning-based parameter optimization mechanisms and an interactive graphical interface. The core of the system consists of functions for modeling neuronal dynamics, calculating the error between real and simulated EEG signals, and generating synthetic EEG data, supplemented by components for working with neural networks, including extracting time and frequency characteristics from EEG signals, automatically generating training data sets, training a neural network, and predicting optimal model parameters (Fig. 1).

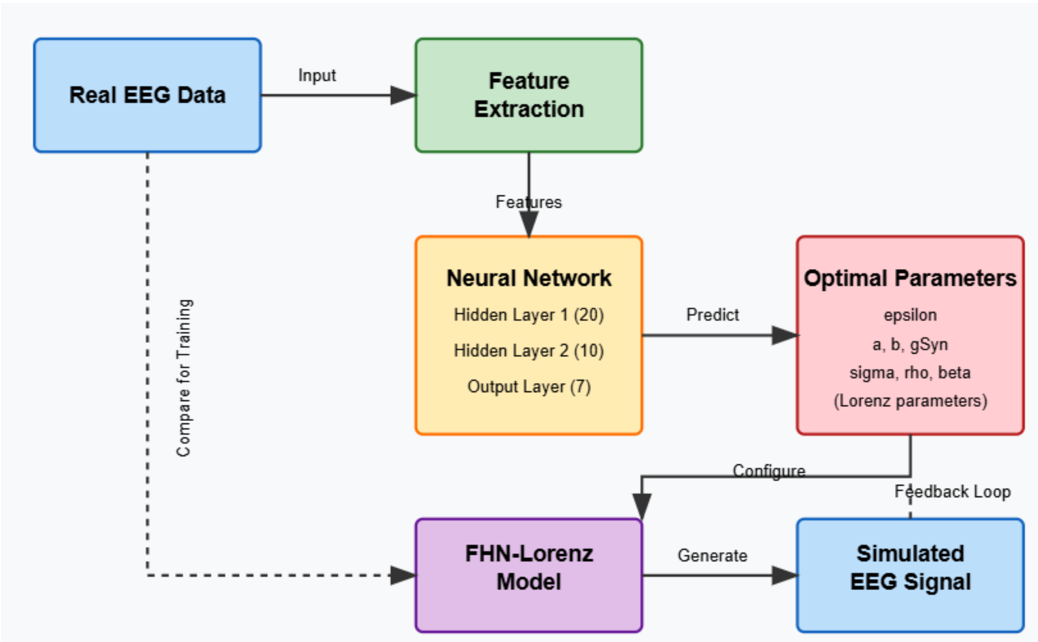

Figure 1 - Structure of the program implementation

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The intuitive graphical interface (Fig. 2) provides interactive control of model parameters via sliders, real-time visualization of real and simulated EEG signals, neural activity, and Lorenz attractor, and provides functionality for loading EEG data in various formats (CSV, MAT, EDF), training a neural network, and exporting results, demonstrating an effective combination of computational neuroscience, machine learning methods, and interactive visualization in a single software solution for researchers and clinicians.

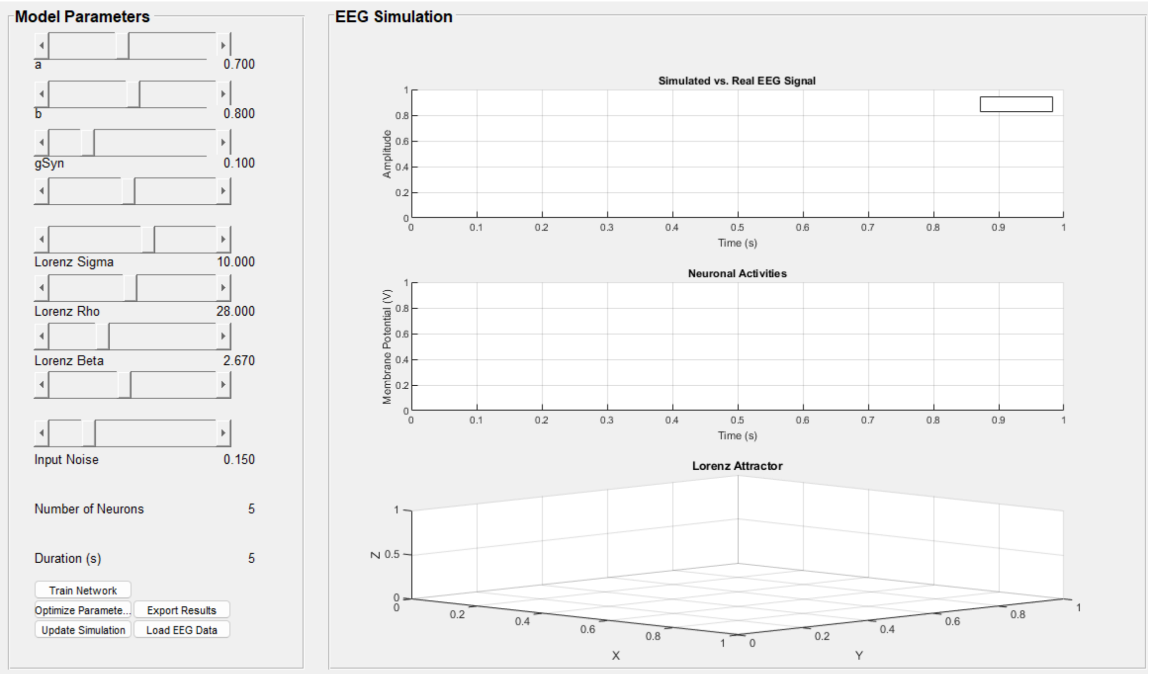

Figure 2 - Graphical interface with interactive elements for EEG modeling

To demonstrate the modeling results, the parameters of different EEG scenarios were determined and the corresponding model was synthesized (Fig. 3).

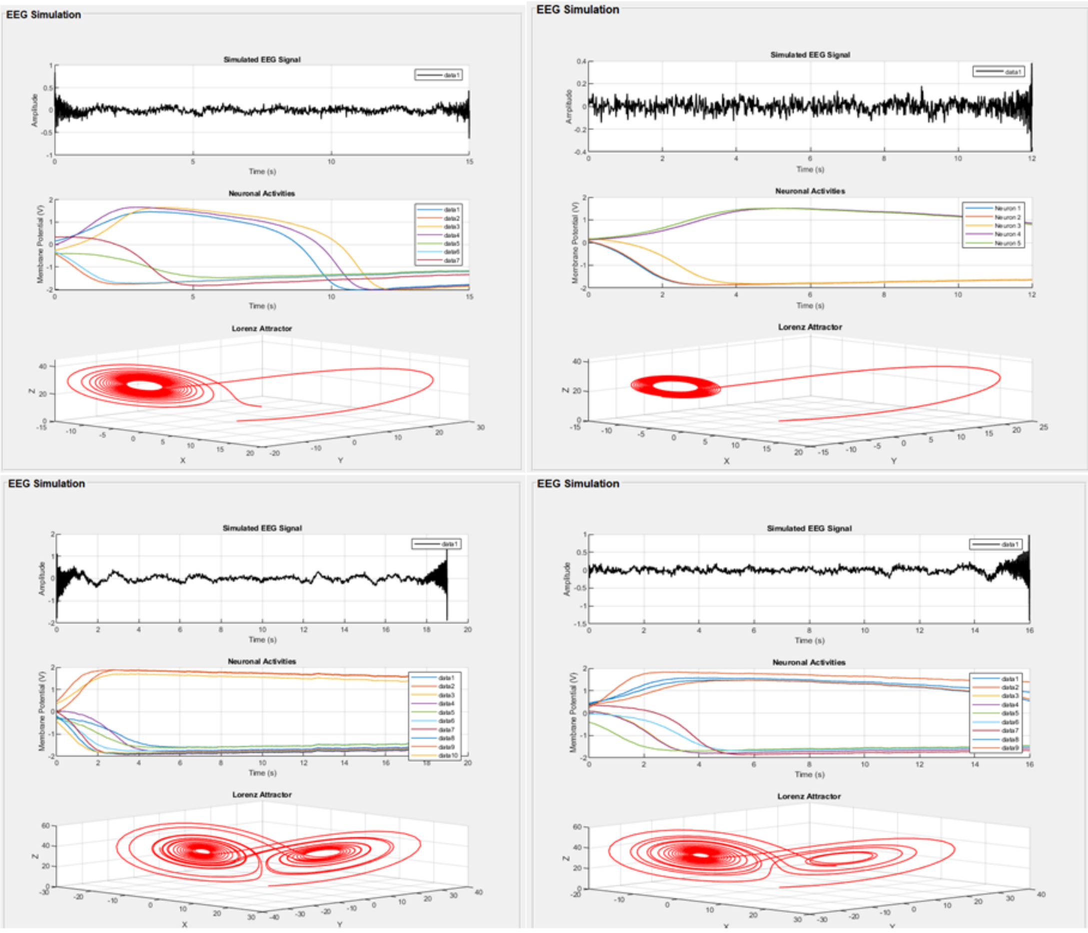

Figure 3 - (a) anxious cognitive processing $\epsilon = 0.1$, $a = 0.75$, $b = 0.85$, $g_{syn} = 0.15$, $\sigma = 10$, $\rho = 30.0$, $\beta = 2.667$; (b) drowsy state $\epsilon = 0.05$, $a = 0.6$, $b = 0.7$, $g_{syn} = 0.09$, $\sigma = 9.5$, $\rho = 25.0$, $\beta = 2.5$; (c) primary epileptic seizure $\epsilon = 0.02$, $a = 0.4$, $b = 0.6$, $g_{syn} = 0.3$, $\sigma = 10$, $\rho = 35.0$, $\beta = 3$; (d) absence $\epsilon = 0.03$, $a = 0.45$, $b = 0.65$, $g_{syn} = 0.25$, $\sigma = 10$, $\rho = 32.0$, $\beta = 2.9$

The presented MATLAB script implements a comprehensive system for processing and analyzing real electroencephalographic (EEG) data using an integrated computational model. The system supports the loading of EEG data in various formats (MAT, CSV, EDF), after which the signal amplitude is normalized to ensure the stability of the simulation, and the loaded data is stored as a time series with an appropriate sampling rate (Fig. 4).

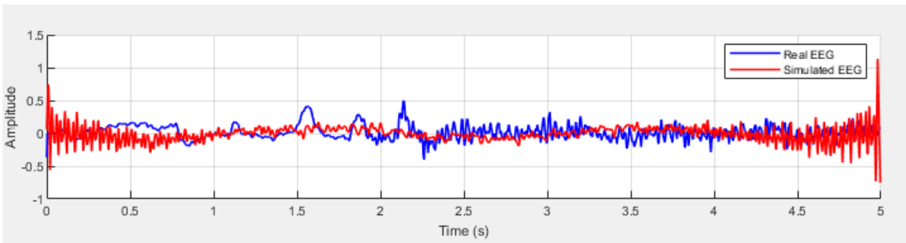

Figure 4 - Uploaded data compared to a synthetic signal without parameter optimization

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Next, a comprehensive analysis of the EEG signal is performed, extracting significant characteristics in both the time domain (mean, standard deviation, variance, skewness, kurtosis, maximum and minimum values, signal range, median, interquartile range, median absolute deviation) and frequency domain (power in key EEG bands: delta, theta, alpha, beta, gamma; relative power in each band; ratio of powers between different bands), as well as nonlinear characteristics such as approximate entropy to detect signal complexity and regularity.

To optimize the model parameters, a feed-forward neural network is used (Fig. 5), which has a multilayer architecture with two hidden layers (20 and 10 neurons, respectively), normalization of input characteristics and output parameters, a maximum number of training epochs of 1000, a minimum gradient of 1e-7, a maximum number of validation runs without improvement of 20, and a data distribution of 70% for training, 15% for validation, 15% for testing. The process of generating training data for the neural network divides the input EEG data into separate segments, for each segment generates random combinations of model parameters in certain ranges, simulates the model with real EEG as an input signal, calculates the root mean square error between the simulated and real EEG, creates pairs of "EEG characteristics - optimal parameters" for training the neural network, and duplicates the best parameters in the data set to increase their significance.

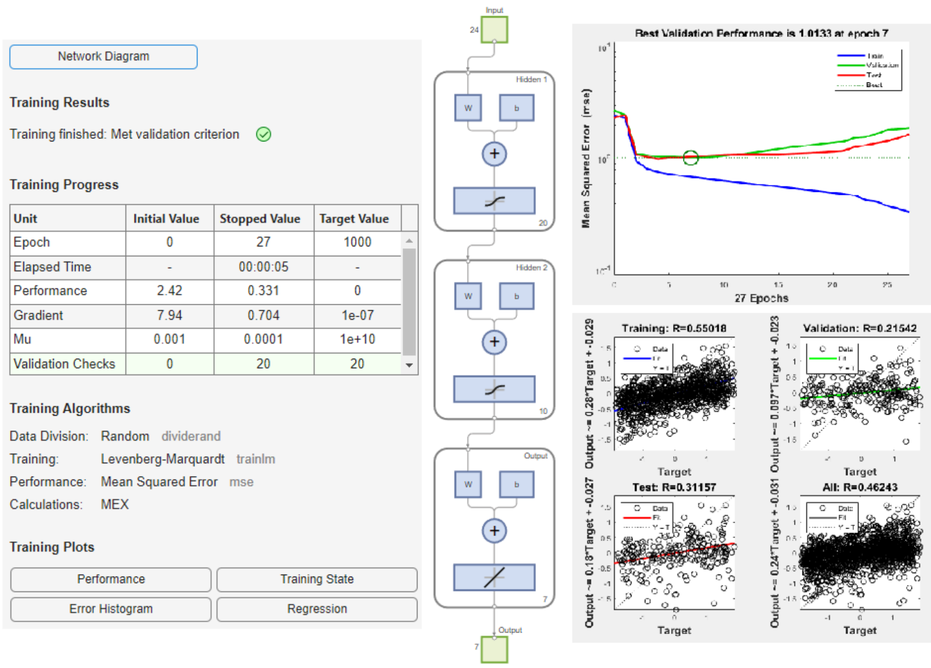

Figure 5 - Structure of the neural network

The next step is to use the trained neural network to predict the optimal model parameters based on the characteristics of the new EEG signal by extracting the characteristics, normalizing these characteristics using the parameters stored during training, passing the normalized characteristics through the neural network, denormalizing the output parameters to obtain real values, and generating a structure with the optimal model parameters. The neural network optimizes seven key parameters of the FHNL model: ϵ, a and b (define the dynamics of the neuron), g_{syn} (the strength of synaptic connection between neurons), and σ, ρ, β (parameters of the Lorentz system that define chaotic dynamics). After optimizing the parameters, the system runs a simulation with the optimized parameters and evaluates the quality of the simulation through the root mean square error between the real and simulated EEG, correlation coefficient, and visual comparison on graphs, allowing you to export simulation results, model parameters, and quality metrics for further analysis and use (Fig. 6).

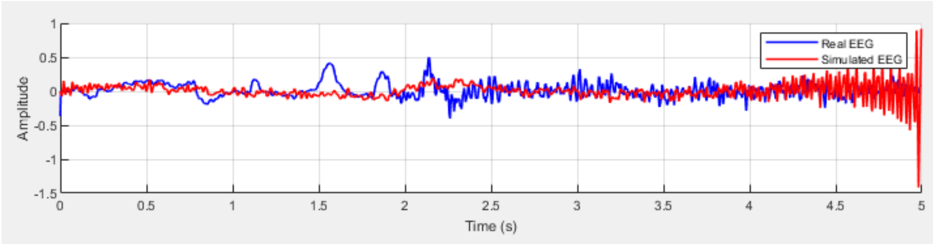

Figure 6 - Downloaded data with a synthetic signal after parameter optimization,
$\epsilon = 0.396, a = 0.899, b = 0.1, g_{syn} = 0.094, \sigma = 20, \rho = 10, \beta = 5$

Thus, the presented script implements the full cycle of EEG data processing from downloading to optimized modeling using neural network technologies to automatically select the optimal parameters of the FHNL model, which allows to effectively simulate neural activity and reproduce real EEG signals with a certain accuracy and correspondence to the original data.

Conclusions. 1. This paper presents a new approach to modeling EEG signals using a hybrid FitzHugh-Nagumo-Lorenz model and parameter optimization based on machine learning. The developed model demonstrates a high ability to reproduce the characteristic features of real EEG signals, while maintaining the biophysical basis and interpretability.

2. The proposed neural network approach to parameter optimization greatly simplifies and speeds up the process of model tuning for specific EEG data. An interactive software tool developed in MATLAB provides a user-friendly interface for researchers and clinicians.

3. The results indicate the potential of this approach for both basic neuroscientific research and clinical applications in the diagnosis and modeling of pathologies. Further research will be aimed at expanding the model's capabilities and adapting it for specific practical tasks.

4. The Lorentz system made it possible to take into account some dynamic aspects of the behavior of signals arising in the human cerebral cortex. Therefore, one of the next tasks will be to study a new hybrid model, the chaotic behavior of which will be set by the systems presented in the works [8, 9].

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Гібридне моделювання ЕЕГ: модель Фіцхью-Нагумо-Лоренца

У статті представлено новий метод моделювання електроенцефалографічних (ЕЕГ) сигналів за допомогою гібридної біофізичної моделі, яка поєднує динаміку Фіцхью-Нагумо з хаотичною системою Лоренца. Розроблено комплексний підхід до оптимізації параметрів моделі на основі нейронних мереж, що дозволяє автоматично налаштовувати параметри для максимальної відповідності реальним ЕЕГ даним. Запропонована модель демонструє здатність відтворювати характерні особливості сигналів ЕЕГ, включаючи основні ритми та відповідні спектральні характеристики. Інтерактивний програмний інструмент, розроблений в середовищі MATLAB, забезпечує зручний інтерфейс для дослідників та клініцистів. Отримані результати свідчать про потенціал цього підходу як для фундаментальних нейронаукових досліджень, так і для клінічного застосування в діагностиці та моделюванні нейропатологій.

Ключові слова: ЕЕГ, FHN, FHNL, система Лоренца, нейронна мережа, моделювання, оптимізація параметрів, MATLAB.

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