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CLASSIFICATION OF EYE STATE BASED ON EEG DATA USING RECURRENCE ANALYSIS

Abstract. The relevance of this study is driven by the growing interest in portable EEG devices and the need to develop efficient algorithms for analyzing brain activity with limited technical resources. This paper addresses the problem of classifying brain states based on electroencephalography (EEG) data to distinguish between two specific states: relaxation and concentration. The classification of open and closed eyes is examined, as eye closure is associated with increased relaxation. A classification method based on the quantitative analysis of recurrence plots, which is one of the approaches of chaos theory, is proposed and compared with traditional brain rhythm analysis. Experimental results showed that the recurrence analysis method outperforms spectral analysis in classification accuracy, particularly for the O1 point, where accuracy increased from 86% to 95%. The optimal parameters for phase space reconstruction were determined: delay 25 ms and dimension of the embedding space 4, which are consistent with the spectral characteristics of the signal. Feature importance analysis revealed that the most significant parameters for classification are entropy, the length of white vertical and diagonal lines in recurrence plots, as well as determinism and laminarity. The obtained results may be useful for developing EEG analysis algorithms in portable devices and applications in the fields of brain-computer interfaces and cognitive training.
Keywords: EEG classification, open and closed eyes, recurrence analysis, recurrence plots, chaos theory, brain rhythms, phase space, spectral analysis, SVM, determinism.

Statement of the problem. This paper addresses the problem of classifying brain states based on EEG, specifically distinguishing between relaxation and concentration. The classification of open and closed eyes is examined, as eye closure is typically associated with increased relaxation. Therefore, analyzing this phenomenon may contribute to a better understanding of relaxation mechanisms.

Recently, an increasing number of portable EEG devices, such as MyndPlay and Muse [1], have emerged. These devices are relatively low-cost, making them accessible to a wide range of users. This creates a demand for data processing algorithms capable of operating with fewer electrodes and lower-quality signals compared to full-scale medical equipment.

Processed brain state data can be useful for self-monitoring, allowing users to analyze how their habits affect brain activity. This may contribute to increased productivity or improved rest quality. Additionally, such technologies can help individuals train their ability to

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consciously adjust their state according to situational needs, for example, enhancing concentration during work or facilitating relaxation during rest.

A promising direction is also the use of such technologies for controlling devices "by the power of thought," which can be especially useful for people with paralysis.

This article explores the classification of brain states using chaos theory methods, particularly through the analysis of recurrence plots. Additionally, this approach is compared with classification based on brain rhythm analysis, a classical method of EEG signal processing.

Analysis of the latest research and publications. Currently, there are several main approaches to EEG signal processing. The first approach is based on brain rhythm analysis using spectral analysis, which was examined in a previous study [2]. That study focused on signal classification based on the alpha rhythm. The essence of this approach is to calculate the amplitudes of each signal frequency, after which the power of the corresponding frequency bands is determined. These bands are known as rhythms: delta rhythms (0.5–4 Hz), theta rhythms (4–8 Hz), alpha rhythms (8–12 Hz), beta rhythms (12–30 Hz), and gamma rhythms (>30 Hz). As demonstrated in the cited work, the alpha rhythm contains the most information regarding the state of open or closed eyes.

An alternative approach is based on the application of chaos theory. Since brain signals are quasi-periodic, this method also demonstrates effective results. The study by Farzad [3] reviews 55 studies on the application of chaos theory to EEG analysis. The author states: *"The evidence from 55 articles suggests that cognitive function is more frequently assessed than other brain functions in studies using chaos theory. The most frequently used techniques for analyzing chaos include the correlation dimension and fractal analysis. Approximate, Kolmogorov and sample entropy account for the largest proportion of entropy algorithms in the reviewed studies."* This work emphasizes that the choice of delay and dimension of the embedding space are critical parameters that significantly affect the analysis results.

A notable study by Kusuma Mohanchandra [4] examines theoretical approaches of chaos theory to EEG data analysis for brain-computer interfaces (BCI). The study states that correlation dimension is widely used as a quantitative parameter for describing attractors and has proven effective in characterizing brain dynamics at different sleep stages. Moreover, this parameter is applied to differentiate between normal subjects and patients with pathologies such as epilepsy, Alzheimer's disease, dementia, and Parkinson's disease.

Another important study [5] focuses on the use of various approaches to three-dimensional phase space reconstruction for five different brain activity states. EEG signals corresponding to these states in seven subjects were analyzed using methods such as determinism, phase graph, power spectrum, approximate entropy, correlation dimension, and Lyapunov exponents. Although each method has its advantages and limitations, the results confirm the nonlinear dynamic nature of brain activity. In particular, determinism analysis showed that the EEG signal occupies an intermediate position between a random and a deterministic process, indicating a possible chaotic nature of brain activity.

The study by Furman [6] proposes a combined approach of short-time Fourier Transform and recurrence quantitative analysis to improve classification accuracy (STFT-RQA) and compares it with classification based on time-delay embedding and recurrence quantitative parameters (TD-EMB-RQA). The STFT-RQA method demonstrated high efficiency, achieving an overall accuracy of 88.2%. Parameter optimization increased accuracy to 95.9% when using 194 selected features. At the same time, TD-EMB-RQA classification proved less effective than STFT, as the analysis of RQA features did not reveal significant discriminative ability, yielding an average accuracy of 80%. However, the removal of redundant components in the cross-validation scheme improved accuracy to 89.2% for 80 selected parameters.

Studies conducted in this and previous works have highlighted the importance of the 10 Hz frequency for the closed-eye state, which can be considered a fundamental frequency of brain activity. This phenomenon was examined in detail by Garcia-Rill [7], where it is explained that 10 Hz represents the brain's natural "idling" frequency in a resting state. A decrease in this frequency may lead to impaired sensorimotor function, as perception at frequencies below 10 Hz becomes less effective. Conversely, an increase in the alpha rhythm frequency to the beta or gamma range occurs when the reticular activating system (RAS) is activated in response to sensorimotor stimulation. Indirect evidence from fMRI studies of cerebral blood flow supports this concept, as alpha rhythms (particularly the occipital and mu rhythms) correlate with reduced cerebral blood flow, indicating lower brain activity.

Objective. The aim of this article is to develop an algorithm that classifies the given dataset and evaluates its accuracy in comparison with the results obtained through spectral analysis. Special attention is given to a qualitative analysis of the obtained data, which will help identify the most influential parameters of recurrence analysis in the classification process.

Additionally, the study explores the nature of the relaxed state, particularly by analyzing the key characteristics of signals. Identifying the most significant parameters and their impact on the analysis results will provide a deeper understanding of the underlying mechanisms of the system's state and contribute to improving classification approaches for similar data in the future.

Presentation of the main material of the research. The *EEG Motor Movement/Imagery Dataset* [8] was used for analysis. It contains over 1,500 EEG recordings lasting from one to two minutes, collected from 109 participants. During the experiment, participants performed various motor and imagery tasks while 64-channel EEG signals were recorded using the *BCI2000* system [9].

Each participant underwent 14 experimental sessions, including two baseline trials (one-minute each) - one with open eyes and the other with closed eyes. Additionally, three two-minute trials were conducted for each of the four tasks. Participants were instructed to open and close their fists in response to a target appearing on the left or right side of the screen or to imagine performing this movement. Furthermore, additional tasks required them to open and close both fists or both feet, depending on the target position.

The recordings are presented in the *EDF+* format, containing 64 EEG signals recorded at a frequency of 160 Hz, along with an annotation channel. The annotations include codes indicating different states: *T0* for rest, *T1* for the initiation of movement (real or imagined) of the left fist or both fists, and *T2* for the movement of the right fist or both legs. The recordings

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were obtained using electrodes based on the international *10-10* system, which excludes certain specific electrodes. The *EDF+* format is adapted for use with the *PhysioToolkit* software [10].

Each participant has a unique identifier ranging from *S001* to *S109*, along with 14 files labeled *R01–R14*, containing the corresponding EEG recordings. Thus, each recording file is represented in the format *SsssRrr.edf*, where s denotes the participant number and r represents the experiment stage. In this study, recordings from the following stages were used:

- 1 – resting state, eyes open;
- 2 – resting state, eyes closed.

Signal Normalization. Preprocessing of the signal is critical for recurrence analysis. In particular, noise removal is an important step, as demonstrated in previous studies. In this experiment, the following types of noise were eliminated: low-frequency noise (<2 Hz), which may reflect head movements and blinking; 50/60 Hz noise caused by interference from the AC power grid; and high-frequency noise from muscle activity (>50 Hz). To achieve this, the Short-Time Fourier Transform (STFT) was applied using a Hann window with a segment size of 1 second [2].

The next important step is signal normalization, which allows data obtained from different electrodes, participants, and time points to be brought into a unified numerical range. In Rolink's study [11], Z-score normalization was applied for electrocardiogram processing, as well as normalization of 30-second segments based on standard deviation.

In this study, Z-score normalization with 1-second segments was used for EEG signal processing. To ensure the continuity of the normalized signal, the segments were merged using a Hann window.

The Hann function [12] is defined as:

$$w[n] = 0.5\ \left[1 - \cos\left(\frac{2\pi n}{N}\right)\right] = \sin^2\left(\frac{\pi n}{N}\right), \quad 0 \le n \le N.$$

A key feature of this window is that the sum of the function and its copy, shifted by half the window size $\frac{N}{2}$, equals one:

$$w[n] + w[n + N/2] = \sin^2\left(\frac{\pi n}{N}\right) + \sin^2\left(\frac{\pi\left(n + \frac{N}{2}\right)}{N}\right) =$$

$$= \sin^2\left(\frac{\pi n}{N}\right) + \cos^2\left(\frac{\pi n}{N}\right) = 1.$$

Thus, if the signal segments overlap by half, following the scheme

$$[0,2H], [H,3H], [2H,4H], [3H,5H]\ldots, \quad H = \frac{N}{2},$$

then their recombination results in a restored signal without distortions. This processing technique is known as windowing.

Since the signal recording has finite boundaries, their handling must be considered. The simplest approach is to pad the signal with zero values at the beginning and end, and then remove these values after recombination.

The formula for Z-score normalization [11] is given by:

$$z_i = \frac{x_i - \mu}{\sigma}, \quad i = 0, \dots, N,$$

where x_i is the value of a discrete data array, N is the size of the array, z_i is the normalized value, μ is the mean of the array, and σ is the standard deviation.

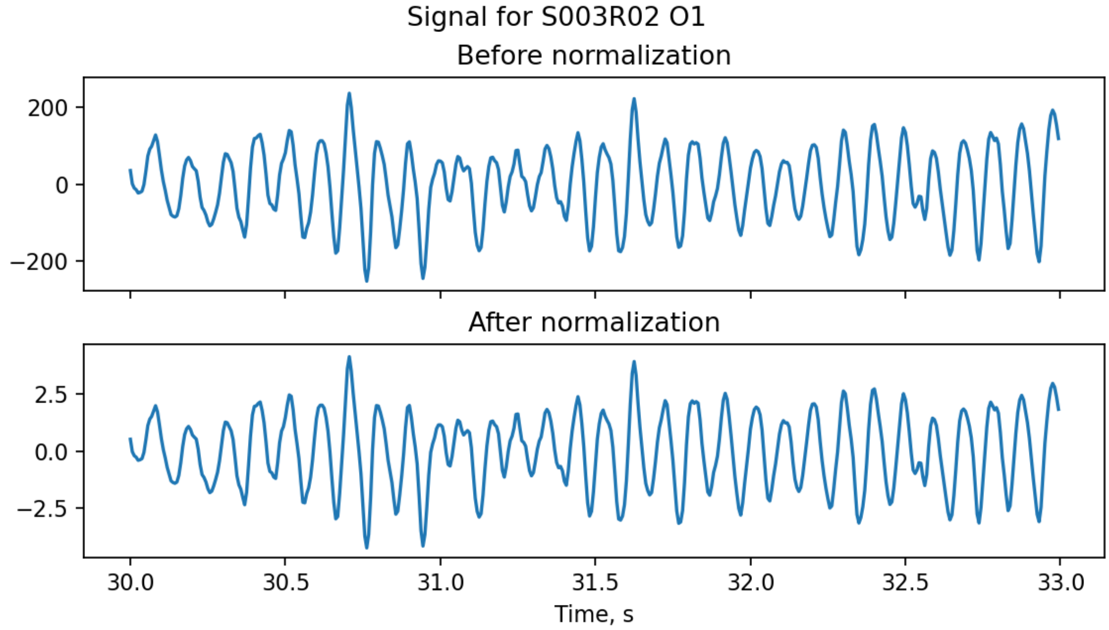

Figure 1 - Signal normalization

Figure 1 shows the normalized signal for the O1 electrode with eyes closed. It can be seen that the signal values after normalization fall within the range $[-3,3]$. In some cases, such as during blinking, the values may reach up to 5, but the overall distribution remains approximately within the range $[-2.5,2.5]$.

Recurrence Analysis. Recurrence analysis is used to study dynamical systems that may exhibit complex and nonlinear behavior. To analyze time series data such as EEG signals, a phase space is constructed. Let $\mathbf{u}_i$ be the time series of the EEG signal. To represent it in the phase space, the method of time delay embedding is used. The embedding vector is defined [13] as:

$$\mathbf{x}_i = \left(\mathbf{u}_i, \mathbf{u}_{i+\tau}, \dots, \mathbf{u}_{i+(m-1)\tau}\right)$$

where τ is the time delay, m is the number of dimensions (embedding), $\mathbf{u}_i$ is the input signal, and $\mathbf{x}_i$ is the resulting point in the m-dimensional phase space. This approach allows for the construction of a multidimensional representation of the original signal.

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Next, to analyze recurrence, the recurrence matrix $R_{i,j}$, $i, j = 1, \dots, m$ is introduced, which determines whether the system returns to one of its previous states:

$$R_{i,j} = \Theta\left(\varepsilon - \| \mathbf{x}_i - \mathbf{x}_j \|\right)$$

where $\| \mathbf{x}_i - \mathbf{x}_j \|$ is the distance between two points $\mathbf{x}_i$ and $\mathbf{x}_j$ in the phase space, ε is the threshold that determines when two points are considered recurrent, and Θ is the Heaviside function, which takes the value 1 if the condition is satisfied, and 0 otherwise.

A recurrence plot is a visualization of the matrix $R_{i,j}$, $i, j = 1, \dots, m$, where recurrent events are shown as black dots. Patterns in this plot help identify features of the signal's dynamics, such as differences between the eyes-open and eyes-closed states.

Additionally, numerical quantitative measures can be computed from the recurrence matrix [14], such as:

• L_{min} (Minimum diagonal line length): the minimum length of a diagonal line in the recurrence plot considered for the calculation of determinism.

• V_{min} (Minimum vertical line length): the minimum length of a vertical line considered for laminarity.

• W_{min} (Minimum white vertical line length): the minimum length of a white vertical line used to calculate trapping time.

• RR (Recurrence rate): a measure of recurrence that indicates the percentage of recurrent points in the matrix:

$$RR = \frac{1}{N^2} \sum_{i,j=1}^{N} R_{i,j}.$$

- DET (Determinism): the proportion of points forming diagonal lines:

$$DET = \frac{\sum_{l=l_{\min}}^{N} l\, P(l)}{\sum_{l=1}^{N} l\, P(l)}.$$

- L (Average diagonal line length): the average length of diagonal lines:

$$L = \frac{\sum_{l=l_{\min}}^{N} l\, P(l)}{\sum_{l=l_{\min}}^{N} P(l)}.$$

- L_{max} (Longest diagonal line length): the length of the longest diagonal line in the plot.
- DIV (Divergence): the reciprocal of the longest diagonal line length:

$$DIV = \frac{1}{L_{\max}}.$$

- L_{entr} (Entropy of diagonal lines): the entropy of diagonal line lengths, which reflects their diversity:

$$L_{entr} = -\sum_{l=l_{\min}}^{N} P(l) \log P(l).$$

- LAM (Laminarity): the proportion of points forming vertical lines:

$$LAM = \frac{\sum_{v=v_{\min}}^{N} v\,P(v)}{\sum_{v=1}^{N} v\,P(v)}.$$

- TT (Trapping time): the average length of vertical lines:

$$TT = \frac{\sum_{v=v_{\min}}^{N} v\,P(v)}{\sum_{v=v_{\min}}^{N} P(v)}.$$

- V_{max} (Longest vertical line length): the length of the longest vertical line in the plot.
- V_{entr} (Entropy of vertical lines): the entropy of vertical line lengths.
- W (Average white vertical line length): the average length of white vertical lines (gaps between recurrent events).
- W_{max} (Longest white vertical line length): the length of the longest white vertical line.
- W_{div} (Longest white vertical line divergence): the reciprocal of the longest white vertical line.
- W_{entr} (Entropy of white vertical lines): the entropy of white vertical lines.
- $\frac{DET}{RR}$ (Ratio of determinism to recurrence rate): the ratio of determinism to recurrence rate.
- $\frac{LAM}{DET}$ (Ratio of laminarity to determinism): the ratio of laminarity to determinism, indicating the proportion of trapped states in the system.

Here:

- N is the number of points in the analyzed time series, or the number of points in the phase space forming the recurrence matrix.
- l is the length of a diagonal line in the recurrence plot. It corresponds to the number of consecutive recurrent points forming a diagonal.
- v is the length of a vertical line in the recurrence plot. It corresponds to the number of consecutive recurrent points forming a vertical line.
- $P(l)$ is the number of diagonal lines of length l in the recurrence plot.
- $P(v)$ is the number of vertical lines of length v in the recurrence plot.

Determining the parameters τ and m is a crucial step in the reconstruction of the phase space. There are several approaches for selecting the optimal value of the delay τ, with two main methods being: the mutual information (MI) method and the autocorrelation method [15].

The MI method is used to analyze the interdependence between the values of the time series and its time-shifted version. However, in the case of quasi-periodic series with harmonics of decreasing amplitude, this approach may yield incorrect results, as the choice of τ may be based on insignificant high-frequency components.

The autocorrelation method is a more traditional way of determining τ and is based on calculating the correlation coefficient between the original signal $x(t)$ and its delayed copy by τ. This approach was introduced in the book [16]. The value of τ is typically chosen at the

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point where the autocorrelation coefficient ρ drops to the level of $1/e^{24}$. The most common types of autocorrelation used are Pearson correlation and Spearman correlation.

There are various methods for determining the dimension of the embedding space, with the most common being the box-counting method, the correlation dimension method, and the false nearest neighbors (FNN) method.

The false nearest neighbors (FNN) method is one of the most widely used approaches for geometrically determining the minimal dimension of the embedding space n when reconstructing the state space [15]. It is based on the idea that, in an insufficiently dimensional embedding space, neighboring points may appear close to each other due to the projection of a higher-dimensional space onto a lower-dimensional one. The method allows determining the optimal n at which the relative number of false neighbors becomes minimal.

To apply this method, the time series is repeatedly embedded into a sequence of m-dimensional Euclidean spaces over a range of increasing values of m. The core idea is that once the minimum dimension of the embedding space m is reached (i.e., $m \geq n$), the distance between neighboring points does not change significantly with further increases in m. In other words, the Euclidean distance $d_m(i,j)$ between a point $P_i \in \mathbb{R}^m$ and its nearest neighbor $P_j \in \mathbb{R}^m$ changes minimally when the dimension of the embedding space increases to $m+1$.

If the dimension of the embedding space m is too small, then the points are considered false neighbors if their pairwise distance increases significantly when m is incremented. This change in distance between nearest neighbors embedded in $\mathbb{R}^m$ and $\mathbb{R}^{m+1}$ is quantitatively assessed using the false nearest neighbors ratio:

$$R_i = \sqrt{\frac{d_{m+1}^2(i,j) - d_m^2(i,j)}{d_m^2(i,j)}}.$$

Next, R_i is compared to a tolerance threshold R_{tol} to distinguish false neighbors, considering them false when $R_i > R_{tol}$. In this study, we choose $R_{tol} = 15$. By applying this threshold to all points, we can compute the percentage of false nearest neighbors, FNN P_{FNN}.

If the system is noise-free, P_{FNN} should reach zero once a sufficient dimension of the embedding space is achieved. However, in the presence of additional noise, P_{FNN} may never reach zero. Therefore, it is common practice to use a cutoff percentage for FNN to determine a sufficient dimension of the embedding space n. We use the commonly adopted threshold $P_{FNN} < 10\%$, which is suitable for most applications involving moderate noise.

Both methods are implemented in the Teaspoon mathematical package for the Python programming language for signal processing [17], which was chosen for use in the experiments.

During the experiments, it was found that determining the parameters τ and m without windowed normalization led to ambiguous results. However, after normalization, it became possible to clearly identify the values of these parameters. When analyzing EEG signals in the eyes-closed state across different participants, it was determined that the dimension of the embedding space is $m = 4$ and the time delay is $\tau = 4$, which corresponds to 25 ms ($\tau = 4/160$), based on a sampling rate of 160 Hz. Figure 2 shows the computed values of τ and m for all recordings.

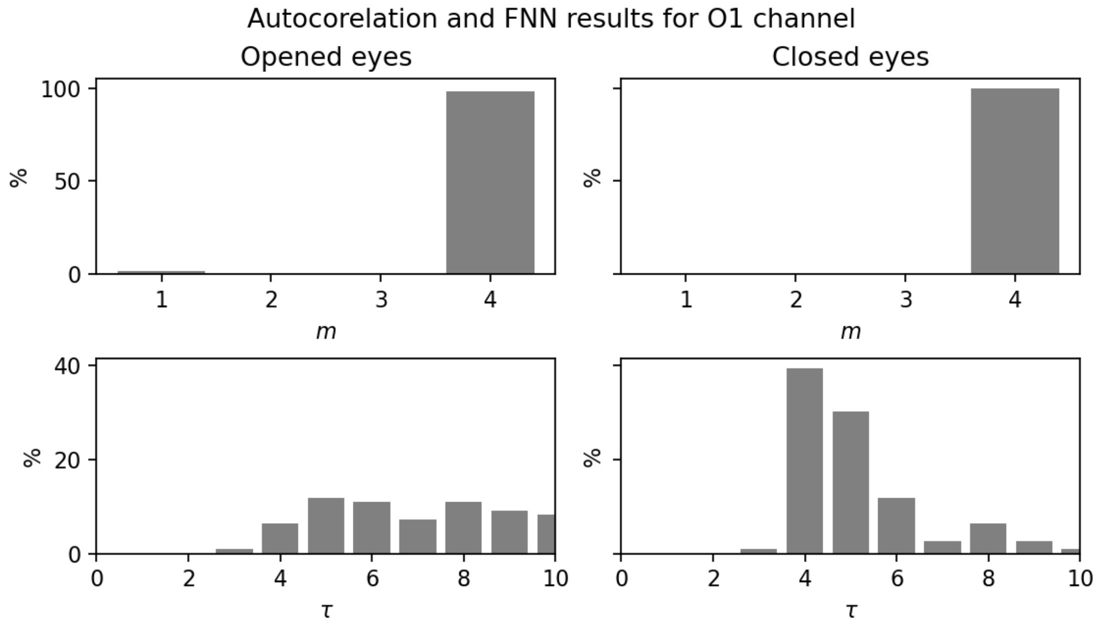

Figure 2 - Calculated τ and m for dataset recording

These results are consistent with spectral analysis, where a peak in power at 10 Hz is observed in the eyes-closed condition. This indicates that the system returns to approximately the same state every 100 ms. Thus, in the eyes-closed state, brain dynamics can be represented as periodic motion in a 4-dimensional space with a time delay of 25 ms, forming a complete cycle over 100 ms. This confirms that the chosen parameters align with the physiological characteristics of brain activity.

Recurrence Plots. To construct recurrence plots, the PyRQA module [18] was used, which provides optimized computational efficiency. PyRQA is based on OpenCL technology, allowing the use of a GPU to accelerate mathematical operations through parallel execution. This significantly reduces computation time compared to traditional approaches.

According to the study [19], using OpenCL speeds up the construction of recurrence plots by more than 5 times compared to OpenMP, which utilizes all CPU cores. When compared to single-core execution, the speedup can reach up to 28 times. This makes PyRQA an

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efficient tool for analyzing large EEG datasets, which is crucial for studying the complex dynamics of brain signals.

Analysis of recurrence plots for eyes-open and eyes-closed states revealed significant differences in the behavior of the brain’s dynamical system. The eyes-open state is characterized by a chaotic structure or a focus on a single state, indicating more complex and variable activity. In contrast, the eyes-closed state shows long diagonal lines, which indicate periodicity in the process. Frequency spectrum analysis confirmed that this periodicity corresponds to 10 Hz, which aligns with the concept of the 10 Hz rhythm as the brain’s “idling” state. An example of this phenomenon is presented in Figure 3.

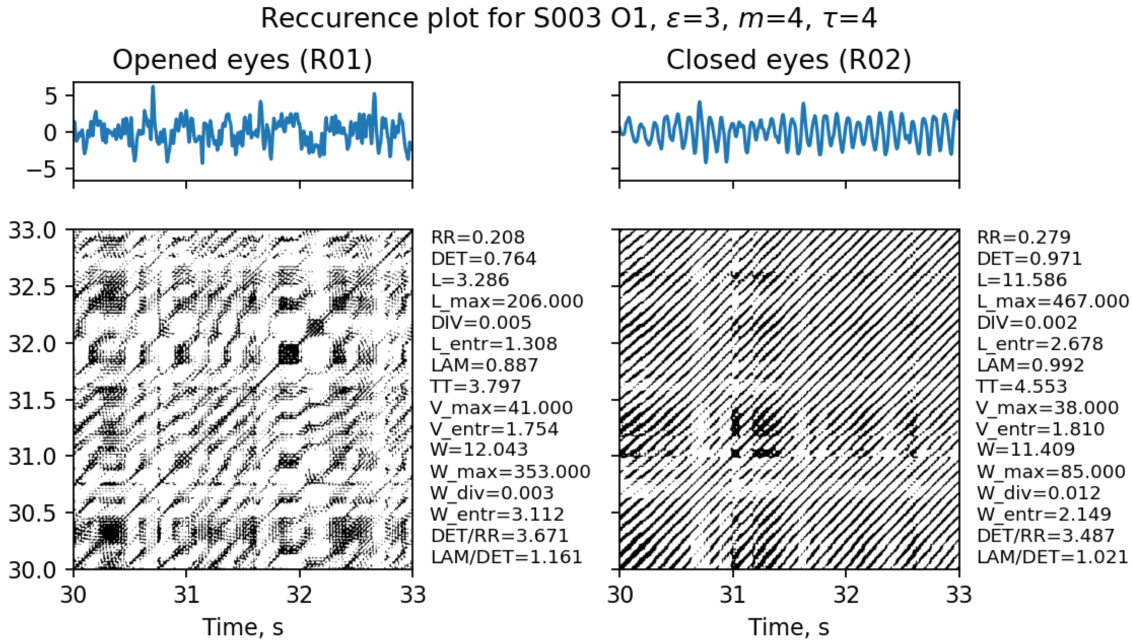

Figure 3 - Recurrence plot

Recurrence Quantitative Analysis. To analyze the obtained recurrence plots, quantitative measures were computed using the PyRQA module. Calculations were performed for all 109 recordings in the dataset, separately for the eyes-open and eyes-closed states, as well as for the Af7, Af8, O1, and O2 electrode locations. The data were segmented into 2-second intervals to compute time-dependent metrics.

To identify the parameter that best separates the eyes-open and eyes-closed classes, a metric R was introduced. Classification accuracy was evaluated separately for the eyes-open (A_o) and eyes-closed (A_c) cases. The overall classification accuracy was defined as $A = \min(A_o, A_c)$ to ensure balance between the classes.

The mean absolute difference between R_{avg} and R was calculated for each participant using the formula:

$$D_{avg} = \frac{1}{n}\sum_{s=0}^{n} |R_{avg} - R|.$$

This value reflects how much the obtained R values for each participant deviate from the average value R_{avg}.

Analysis of Table 1 shows that the most informative features for classifying the eye state (open or closed) are determinism (DET), entropy of vertical lines (V_{entr}), average length of white vertical lines (W), and longest white vertical line divergence (W_{div}).

These recurrence analysis parameters make it possible to distinguish the structure of time series in eyes-open and eyes-closed states. In particular, high DET values indicate greater regularity and predictability in signal dynamics, which is characteristic of the eyes-closed state. Similarly, the entropy of vertical lines (V_{entr}) reflects the variability of transitions between states, which tends to be more chaotic with eyes open.

Thus, the obtained results confirm that recurrence analysis can be an effective approach for automatic classification of eye state based on EEG signals.

Table 1

results of a simple classification of open and closed eyes

Parameter	Point	A	A_o	A_c	R_{avg}	D_{avg}
DET	Af7	60.91%	60.91%	72.73%	0.26	0.04
DET	Af8	61.82%	61.82%	70.00%	0.25	0.04
DET	O1	58.18%	58.18%	58.18%	0.23	0.01
DET	O2	55.45%	55.45%	62.73%	0.23	0.01
V_{entr}	Af7	66.36%	66.36%	70.91%	54.77	15.29
V_{entr}	Af8	60.00%	60.00%	73.64%	54.11	14.85
V_{entr}	O1	58.18%	58.18%	68.18%	45.81	5.28
W	O1	55.45%	55.45%	55.45%	1.95	0.04
W	O2	55.45%	55.45%	55.45%	1.95	0.05
W_{div}	Af7	69.09%	77.27%	69.09%	407.38	48.20
W_{div}	Af8	70.00%	80.00%	70.00%	399.70	53.55

Classification Using Support Vector Machine. The Support Vector Machine (SVM) method with C-support vector classification [20] was applied to improve the accuracy of eye state classification. The RBF (Radial Basis Function) kernel was used, enabling effective handling of nonlinearly separable classes. The model was configured with parameters $C = 100$ and $\gamma = 1$, which control the classifier's flexibility and the scale of influence of individual data points.

The implementation was carried out using the scikit-learn library [21], which utilizes LIBSVM. This ensures efficient optimization of the separating hyperplane and improves classification accuracy compared to the threshold-based method using recurrence features.

Eye state classification (open or closed) was initially performed based on brain rhythms divided into frequency bands: delta (0–4 Hz), theta (4–8 Hz), low alpha (8–10 Hz), high alpha

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(10–12 Hz), low beta (12–16 Hz), mid beta (16–20 Hz), high beta (20–30 Hz), low gamma (30–40 Hz), and mid gamma (40–50 Hz).

For classifier training, the first half of each participant's recording was used, and the second half was used for testing. The resulting classification accuracies for different electrodes were: Af7 – 89.38%, Af8 – 87.40%, O1 – 86.80%, O2 – 86.51%.

Next, recurrence quantitative measures were used for classification, which significantly improved the results. Accuracy increased to: Af7 – 94.41%, Af8 – 93.85%, O1 – 95.38%, O2 – 95.77%. This indicates that recurrence features are more informative for classifying eye states compared to brain rhythm frequency analysis.

Thus, classification based on recurrence measures demonstrated higher accuracy, especially in the occipital region of the brain (O1, O2), where primary visual processing takes place. This confirms that recurrence analysis methods more effectively capture patterns associated with changes in eye state than frequency-based approaches.

Thus, classification based on recurrence measures demonstrated higher accuracy, particularly in the occipital region of the brain (O1, O2), where primary visual processing occurs. This confirms that recurrence analysis methods are more effective at detecting patterns associated with changes in eye state than brain rhythm frequency analysis.

Analysis of the influence of individual parameters using the SHAP module (Figure 4) showed that the most significant features for classification are the inverse of the longest white vertical line (W_{div}), the entropy of white vertical lines (W_{entr}), the average diagonal line length (L), the longest white vertical line (W_{max}), the longest vertical line in the plot (V_{max}), and laminarity (LAM). This indicates that the structure of white vertical and diagonal lines in recurrence plots plays a key role in recognizing the eye state.

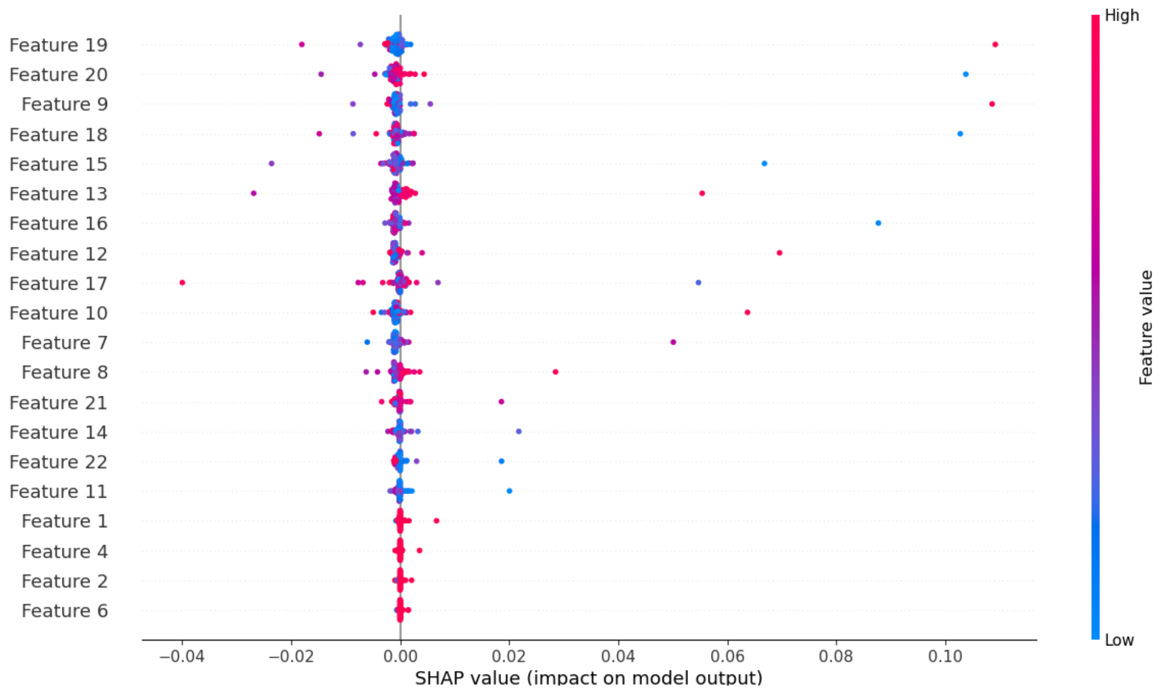

Figure 4 - Determining the most influential classifier parameters

Conclusions. Thus, the use of quantitative recurrence measures for eye state classification proved to be more effective than traditional brain rhythm analysis. For the O1 electrode, classification accuracy based on recurrence features reached 95%, compared to 86% when using brain rhythms. This confirms that methods from chaos theory may be more effective for analyzing quasi-periodic brain signals.

The optimal minimum segmentation length for the signal is one second. Smaller segments result in the loss of useful information. At the same time, the complete absence of segmentation makes the signal more susceptible to low-frequency noise, which distorts its dynamics, increases the system's dimensionality, and significantly degrades the quality of the recurrence plots.

The delay τ used for phase space reconstruction is 25 ms, and the dimension of the embedding space m is 4. This is consistent with the results of spectral analysis, where a peak at approximately 10 Hz is observed in the eyes-closed condition. Since the period of oscillations at this frequency is 100 ms, the chosen delay $\tau = 25$ ms corresponds to one-quarter of the cycle, which is optimal for accurate phase space reconstruction. Therefore, the selected values of τ and m are well-justified. In future work, we will attempt to construct a system of differential equations that describes the chaotic behavior of signals in the human cerebral cortex. Studying the attractors of such a system using the methods proposed in [22] may provide additional insights into brain states associated with relaxation and concentration.

When constructing a simple classifier using optimal threshold search, it was found that the most influential parameters are determinism (DET), entropy of vertical lines (V_{entr}), average length of white vertical lines (W), and longest white vertical line divergence (W_{div}). Analysis of influential parameters in the SVM classifier revealed that the key features for class separation are the inverse of the longest white vertical line (W_{div}), entropy of white vertical lines (W_{entr}), average diagonal line length (L), longest white vertical line (W_{max}), longest vertical line in the plot (V_{max}), and laminarity (LAM). This highlights the importance of various aspects of the structural organization of recurrence plots for accurate differentiation of eye states.

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Класифікація стану очей на основі ЕЕГ-даних з використанням рекурентного аналізу

Актуальність цієї статті зумовлена зростаючим інтересом до портативних ЕЕГ-пристроїв та необхідністю розробки ефективних алгоритмів аналізу мозкової активності за обмежених технічних ресурсів. У цій статті розглядається проблема класифікації станів мозку за даними електроенцефалографії (ЕЕГ) з метою розрізнення конкретних двох станів розслабленості та концентрації. Досліджується класифікація відкритих і закритих очей, оскільки закриття очей асоціюється з підвищеною розслабленістю. Запропоновано метод класифікації на основі кількісного аналізу рекурентних діаграм, що є одним із підходів теорії хаосу, та проведено його порівняння з традиційним аналізом мозкових ритмів. Результати експериментів показали, що метод рекурентного аналізу перевершує спектральний аналіз за точністю класифікації, зокрема для точки O1 точність зросла з 86% до 95%. Визначено оптимальні параметри реконструкції фазового простору: затримка 25 мс і розмірність простору вкладення 4, що узгоджується зі спектральними характеристиками сигналу. Аналіз важливості ознак показав, що найбільш значущими параметрами для класифікації є ентропія та довжина білих вертикальних і діагональних ліній на рекурентних діаграмах, а також детермінізм і ламінарність. Отримані результати можуть бути корисними для розробки алгоритмів аналізу ЕЕГ у портативних пристроях та застосувань у сфері нейроінтерфейсів і когнітивного тренування.

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