

## ALGORITHMS AND METHODS IN DYNAMIC PROBLEMS OF OPTIMAL TRACK PLACEMENT IN THREE-DIMENSIONAL PRINTING

*Abstract. The work is devoted to the study of algorithms and methods for calculating optimal trajectories in 3-D printing in the formulation of a dynamic problem of the theory of optimal set partitioning. The relevance of this task lies in the significant actualization of 3-D printing, both in the manufacture of medical, military and dual-use products. Nowadays, technological solutions for 3-D printing allow users to manufacture parts from children's toys made of plastic to rocket parts made of high-alloy steel. In times of war, 3-D printing became a tool for improving weapons for Ukraine, allowing you to create experimental products in small quantities without resource-intensive research, investigate their effectiveness and applicability in practice, improve and launch them into mass production. This paper considers the mathematical aspects of constructing 3-D printing trajectories, taking into account the limitations put forward by manufacturers in the analytical formulation. This approach allows you to obtain optimal solutions, minimize the time and cost of refining parts, and generally reduce the time and cost of their manufacture. It should be noted that this approach is very relevant in the bowls of time, which is due to the large number of new inventions that are being developed in various areas of human existence.*

*Keywords: mathematical modeling, theory of optimal set partitioning, dynamic problem, 3-D printing, optimal trajectory.*

**Entry.** In today's world, technological advances in the field of 3D printing are of great interest in the scientific and industrial environment. This technology is widely used in various industries, ranging from manufacturing to medicine and architecture. However, despite the many opportunities it provides, production optimization remains an urgent task that requires constant improvement of algorithms and methods for building 3D printing paths.

An essential part of this optimization is the development of optimal trajectories for 3D printing. Every movement of the print head and the rotation of the printer requires time and materials. The main goal of the study is to develop algorithms and methods for optimizing printing trajectories aimed at minimizing the cost of materials and time, which will increase production efficiency and reduce the cost of manufactured products.

The central task is to create effective algorithms for plotting trajectories in real time. This is important for automating production processes and reducing resource costs. Efficient construction of trajectories will not only increase printing speed, but also improve the quality

of manufactured products, which is of great importance for sustainable production and environmental sustainability.

The study involves the analysis of the physical properties of the environment and their consideration in mathematical models. This will contribute to the creation of more accurate and adaptive algorithms that can avoid obstacles and optimize trajectories to reduce energy and time consumption. Also, the development of algorithms and methods for optimizing 3D printing trajectories involves taking into account different types of materials used in the printing process, as well as the specifics of the work of different models of 3D printers.

In particular, developing optimal print paths can open up new opportunities for custom manufacturing, allowing businesses to increase competitiveness and improve customer experience. The results of the study can also find practical applications in industrial processes where 3D printing is used, contributing to the increase in production efficiency and economy.

The application of the developed algorithms and methods to optimize 3D printing paths can have a significant impact on industries such as the aviation and automotive industries, electronics, medicine, education and scientific research. The importance of this work lies in the fact that it contributes to the development of 3D printing technology, increased production efficiency, and reduced material and energy costs, which in turn can lead to a reduction in the cost of manufactured products and the development of environmentally sustainable production.

Thus, this research is of great importance for the further development of 3D printing and its implementation in various industries and science, contributing to the creation of new opportunities and opening up new horizons for scientists and engineers.

Path optimization in 3D printing is an extremely important task aimed at reducing the time it takes to create objects and improve their quality. Current research in trajectory optimization algorithms offers diverse and innovative approaches to address this complex problem.

**Analysis of literary sources.** The paper "Path optimization in 3D printer: Algorithms and experimentation system" [1] examines the use of hybrid algorithms such as Greedy Two Opt and Greedy Annealing. These algorithms are not only based on choosing an effective initial decision through a fast algorithm, but also improve it by further exploring the decision space. The implementation of these algorithms in the system of experiments confirms their high efficiency and properties.

In the paper "An Approach of Path Optimization Algorithm for 3D Concrete Printing Based on Graph Theory" [2], a path optimization algorithm based on graph theory for 3D printing of concrete is proposed. By using the distribution algorithm and the ant colony algorithm, they achieve not only a reduction in printing time, but also a significant improvement in print quality.

The article "An ACO-Based Tool-Path Optimizer for 3-D Printing Applications" [3] uses an ant colony optimization algorithm to improve the solutions generated by the slicer software. Based on a modified ACO, their optimizer allows you to reduce print time and improve visual print quality.

The paper "Path Optimization Along Lattices in Additive Manufacturing Using the Chinese Postman Problem" [4] uses a graphical model of the 3D printing process and solves the

Chinese postman problem to optimize the extruder movement. This approach aims to improve printing efficiency and accuracy.

The paper "Optimizing the Nozzle Path in the 3D Printing Process" [5] solves the problem of the optimal nozzle path in 3D printing using linear programming and heuristic algorithms. Their results indicate a significant reduction in printing time.

The paper "Accelerating 3D Printing Process Using an Extended Ant Colony Optimization Algorithm" [6] discusses the use of an ant colony algorithm to speed up the 3D printing process. The proposed mechanisms allow you to adaptively adjust the number of iterations, which as a result improves ACO performance.

In the paper "Path Optimization For Cooperative Multi-Head 3d Printing" [7] the problem of layer path optimization (LPOP) in 3D printing is defined. The author investigates modified ant colony algorithms that contribute to reducing travel distance and printing time for various 3D models.

**Formal statement of the problem.** The work involves the development and implementation of a systematic approach, in particular algorithms and methods for building and optimizing the printing trajectory for 3D printers. The task is to formulate a mathematical statement of the problem, develop algorithms and methods of partitioning, and create a software product that will provide automated generation of the optimal path for 3D printing of objects presented in STL format.

The main tasks of the work are:

1. Development of a trajectory optimization algorithm
2. Development of a systematic approach to optimizing the print trajectory, taking into account the model parameters and limitations of the printer.
3. Implementation of the software product.
4. Implementation of a software package that will automatically load and analyze STL files of objects to build an optimal trajectory.
5. Visualization of the process.
6. Providing the ability to visualize the figure at different stages of printing for analysis and optimization of the process.
7. G-code generation. Development of a module to generate G-code corresponding to the simulated trajectory, providing standard commands for the 3D printer and ensuring that it is saved to a file.

The purpose of the work is to develop and implement a systematic approach to building and optimizing the trajectory of printing on a 3D printer, creating a software product that will ensure the effective generation of the optimal path for printing objects presented in STL format. The main tasks include the development of a trajectory optimization algorithm, the implementation of software that interacts with 3D models, the visualization of the printing process, the generation of G-code, and the study of the effectiveness of the developed solution.

**Overview of existing tools.** 3D printing is an important technology that is widely used in various industries, from manufacturing to medicine and design. Among the most common technologies used in our time are the following:

1. Fused Deposition Modeling (FDM). FDM is one of the most widely used 3D printing technologies. In this system, the plastic melts and flows out of the print head, forming an object layer by layer. FDM is a relatively affordable and reliable technology, but may have limitations in accuracy and resolution.

2. Stereolithography (SLA). SLA uses ultraviolet radiation to cure liquid resinous material. This technology provides high precision and allows the production of parts with high detail. However, the limited availability of materials and high costs can be disadvantages.

3. Selective Laser Sintering (SLS). SLS uses a laser to fuse the powder, creating a solid object. This technology allows you to make objects with different materials and quite complex geometric shapes. However, it can be costly and require special equipment.

4. Multi-Jet Fusion (MJF). MJF uses technology similar to an inkjet printer, but uses materials made from powdered substrates. It allows you to obtain parts with high speed and accuracy, but requires special materials.

Each of these systems has its own advantages and limitations, making them suitable for a variety of applications. The selection of a specific system should take into account the requirements of a particular project and the availability of resources. An overview of these technologies provides an important basis for further informed conclusions in the study of a systematic approach to the trajectory of printing on a 3D printer.

#### **Maximum print sizes:**

- FDM: Usually limited by the size of the printer countertop, which can affect the ability to produce large parts.

- SLA: Has the ability to produce high-dimensional parts, but is limited by the size of the resin tank.

- SLS: Allows the production of medium and large parts, but is limited by the dimensions of the working chamber.

- MJF: Has great capabilities for the production of medium and large parts.

#### **Printing Accuracy:**

- FDM: Has lower accuracy compared to other systems, usually in the range of 0.1 - 0.2 mm.

- SLA: Provides high accuracy, can reach values less than 0.05 mm.

- SLS: It has good accuracy, especially for complex geometric shapes, within 0.1 mm.

- MJF: Has high accuracy, usually in the range of 0.02 - 0.03 mm.

#### **Print Speed:**

- FDM: Usually has an average speed, layer by layer, which affects the printing time of large objects.

- SLA: Usually takes longer to print, especially with high detail.

- SLS: It has a high printing speed, especially in the production of serial parts.

- MJF: It is characterized by high printing speed, especially in the manufacture of serial objects.

#### **Materials:**

- FDM: A wide variety of materials available, including plastics, rubber, and composites.

- SLA: Uses a variety of resinous materials, ensuring high surface quality.
- SLS: Uses powdered materials such as polymers or metals.
- MJF: Uses thermoplastic materials, ensuring high strength and quality.

**Processor and software:**

- FDM: Easy to use, often used by small businesses and private users.
- SLA: Requires high security standards, uses specialized software.
- SLS: Requires high-precision equipment and software for production.
- MJF: Requires productive equipment and software for production management.

When choosing a specific 3D printing system, it is important to consider these functional characteristics, as they directly affect the suitability of the system to perform specific tasks and solve certain production tasks.

Analyzing the advantages and disadvantages of each 3D printing system is an important part of the study, as it allows a better understanding of the capabilities and limitations of each technology. Below are the advantages and disadvantages of each system considered:

**Fused Deposition Modeling (FDM):**

Advantages:

- Availability and cost of equipment and materials.
- A wide range of materials available, including plastics, rubber, and composites.
- Easy to use and supported by many printing houses.

Disadvantages:

- Lower accuracy and resolution compared to other technologies.
- Limited possibilities for the manufacture of parts with high detail.

**Stereolithography (SLA):**

Advantages:

- High precision and resolution, ideal for details with high detail.
- Ability to work with different types of resins and materials.

Disadvantages:

- High costs for materials and ultraviolet lasers.
- Limited scalability compared to other technologies.

**Selective Laser Sintering (SLS):**

Advantages:

- High printing speed, especially for the production of serial parts.
- Ability to use a variety of powder materials, including polymers and metals.

Disadvantages:

- Consumable system due to the use of expensive powder materials.
- Requires specialized equipment and management.

**Multi-Jet Fusion (MJF):**

Advantages:

- High printing speed and high accuracy.
- Ability to produce serial parts with high quality.

Disadvantages:

- More expensive compared to FDM and other technologies.
- May be restricted by access to certain materials.

Based on the results of the analysis of existing instruments, the following conclusions can be drawn:

**Cost and Availability:**

- FDM is more affordable, but may have limitations in quality.
- SLAs and SLS require significant equipment and material costs.
- MJF, although more expensive, can be efficient for production.

**Quality and Detail:**

- SLA and MJF provide high quality and detail.
- FDM and SLS may have limitations in high detail.

**Speed:**

- MJF and SLS can be faster for production of serial parts.
- SLA may be slower due to high accuracy.

**Overview of existing research methods.** Learning a variety of methods for constructing trajectories on a 3D printer is an essential step in achieving improvements in the field of additive manufacturing. This study aims to understand and optimize the process of forming printhead trajectories and the impact of this process on the quality and performance of 3D printing.

Among the most common classes of three-dimensional printing methods, it is necessary to highlight:

Coordinates-based methods are trajectory construction methods that use coordinates based on determining points in space that should be connected. The following approaches are considered:

- rectilinear trajectory: connecting points with lines, which is most often used to fill the contour of an object;
- Curved trajectory: using arcs and curves to approximate the shape of an object, which can improve appearance and accuracy.

Vector-based methods are methods that use vectors that focus on the movement of the printhead in a specific direction. The following approaches are considered:

Slicing methods: Splitting an object into thin layers to further build a trajectory for each layer;

Methods for using path vectors: Identification of route points that optimally determine the movement of the print head.

Methods that use mathematical models are methods that are based on mathematical models of the shapes and geometry of objects to build accurate trajectories. The following approaches are considered:

- continuous curves and surfaces: Using mathematical equations to describe the shape of an object and construct smooth trajectories;
- Trajectory optimization techniques: Using optimization algorithms to select the most effective trajectories, taking into account constraints and requirements.

Methods that take into account the peculiarities of the geometry of objects are methods that consist in analyzing how height differences, protrusions and other protruding elements are taken into account.

Object Fill Path Methods are methods for determining the fill path of an object, including fill methods such as "dimensional placeholder" or "hexagonal placeholder".

**Division into layers.** Splitting a three-dimensional model into layers is a process used to decompose an object into successive thin layers along the Z-axis. Let's have an object  $O$  with the volume described by the function  $f(x, y, z)$ , where  $0 \leq z \leq h$ , where  $h$  - the height of the object. Specified layer thickness  $dz$ .

The goal is to define the boundaries of each layer, which will serve as the basis for the generation of further layers. Let's mark  $z_k$  height  $k$ -th layer, where  $z_k = k \cdot dz$ .

Formally, the division into layers can be defined as follows:

$$S_{\downarrow}k = \{(x, y, z) \mid z = z_{\downarrow}k, (x, y) \in D\}, \quad (1)$$

where  $D$  - The plane of the object at the level  $z = z_k$ , which can be obtained by the intersection of the object with the plane  $z = z_k$ . This intersection can be represented as an area  $D \subset R^2$  using parameterization  $D(u, v)$ , where  $u$  and  $v$  plane parameters,  $z = z_k$ .

$$S_{\downarrow}k = \{(u, v, z_{\downarrow}k) \mid (u, v) \in D\} \quad (2)$$

This allows you to get the boundaries of each layer  $S_k$  based on the object's parameterization and layer height. Once the layer boundaries are obtained, they can be used for further processing and path generation.

Consider a general example where a figure is given by a function  $f(x, y, z)$ , which describes the object, and the height of the layer  $z_k$ . Let's denote the parameterization of the plane at the level  $z = z_k$ , as  $D(u, v)$ , where  $u$  and  $v$  Parameters on the plane  $z = z_k$ .

Thus, layer boundaries  $S_k$  can be determined using parameterization  $D(u, v)$  and height  $z_k$ :

$$S_{\downarrow}k = \{(u, v, z_{\downarrow}k) \mid (u, v) \in D\} \quad (3)$$

For example, consider the parameterization of a plane  $D(u, v)$  in the form of a rectangle from 0 to 1 on both axes  $u$  and  $v$ :

$$D(u, v) = \{(u, v) \mid 0 \leq u \leq 1, 0 \leq v \leq 1\} \quad (4)$$

Let the height of the layer  $z_k = 0.1$ . Then the layer boundaries  $S_k$  will:

$$S_{\downarrow}k = \{(u, v, 0.1) \mid 0 \leq u \leq 1, 0 \leq v \leq 1\} \quad (5)$$

This means that the edges of each layer are a rectangle on a plane  $z = 0.1$ , which can be used for further processing and generation of circuits. Parameters  $u$  and  $v$  can range from 0 and 1, which defines the area on the plane  $z = 0.1$ .

**Circuit generation.** To build the printing path of each layer, an important step is the generation of layer outlines. This process defines the boundaries of the object on a given layer

and determines how the printing extruder must pass these boundaries. Mathematical and geometric principles are used to create points and lines that describe the contours of a layer.

After defining the boundaries, internal and external contours are created. The inner paths define the cut of part of the object, while the outer paths define the outer boundary.

The algorithm for generating internal and external contours was implemented using the Boundary Traversal algorithm. The following is an algorithm for determining internal and external contours based on the object's boundary points.

1. The beginning of the algorithm:

Input: The boundaries of the object on the layer in the form of a sequence of points.

Output: Internal and external contours in the form of a sequence of points.

2. Determination of the direction of the border bypass:

- Choose a point  $P_0$  on the boundary of the object as the starting point.

- Finding the point  $P_1$ , which is closest to  $P_0$  and belongs to the boundary of the object.

- Determine the direction of the bypass, for example, clockwise or counterclockwise.

Bypass the border:

- We start bypassing the border from a point  $P_0$ .

- Finding the next point  $P_{i+1}$ , belonging to the boundary, taking into account the determined direction of the bypass.

- We continue this process until we return to the starting point  $P_0$ .

Separation into internal and external contours:

- when bypassing the border, write the points in the corresponding contour (internal or external);

- control whether the point bypasses the boundary of the object within the previously selected direction.

Filtering Circuits:

- cut off the internal contours, which are completely inside the rest of the contours;

- we leave only the outer and inner contours that separate the object.

Discretizing Paths: The points that make up the paths are sampled to represent them as a sequence of coordinates. This may involve using algorithms that approximate curves to the selected step size. The main goal is to represent the curve in the form of a sequence of coordinates for later use in specific calculations, such as plotting a print path.

1. Input: a contour consisting of a smooth curve represented by the coordinates of its points.

2. Step Size Selection: Selects a specific step size that determines the distance between discrete points on the curve.

3. Sampling process: passing along the path and selecting points with the selected step. That is, by choosing every  $n$ th point on a smooth curve.

4. Creating a Sequence of Coordinates: Forming a new list of points, which are now a discretized version of the original outline.

**Print path optimization.** In the process of printing on a 3D printer, there are several possible costly actions that can affect the time and quality of printing:



**Deceleration:** When the printer needs to change the speed of movement, such as when changing a layer or when printing complex parts, slowing down can occur. Switching from high speed to lower speed may result in longer print times.

**Stopping and Waiting:** This occurs when the printer temporarily stops before starting a new layer or waits for confirmation from the user (for example, when changing the material or inserting an object).

**Retraction:** Moving the filament back into the printer nozzle (retraction) to avoid leakage can affect print time, especially with frequent changes in print direction.

**Travel Moves:** When the print head moves between different areas of the object to be printed, such movements can cause slowdown.

**Temperature Control:** Changing the temperature of the extruder or heater may cause stops or slowdowns to reach the optimum temperature.

**Changing real-time printing settings:** If the user chooses to change some print settings while the printer is running, it may also cause stops or slowdowns.

That is, analyzing what is written, the task arises to build a trajectory in such a way as to minimize stops, decelerations and changes in the movement of printing (spreads). Since the preparatory stages of slicing, determining the boundaries of layers and internal/external contours are quite difficult to calculate programmatically and time-consuming, the task is also complicated by building a simple algorithm that will quickly and optimally build a trajectory.

So, the algorithm for building the optimal trajectory can be represented by the following flowchart:

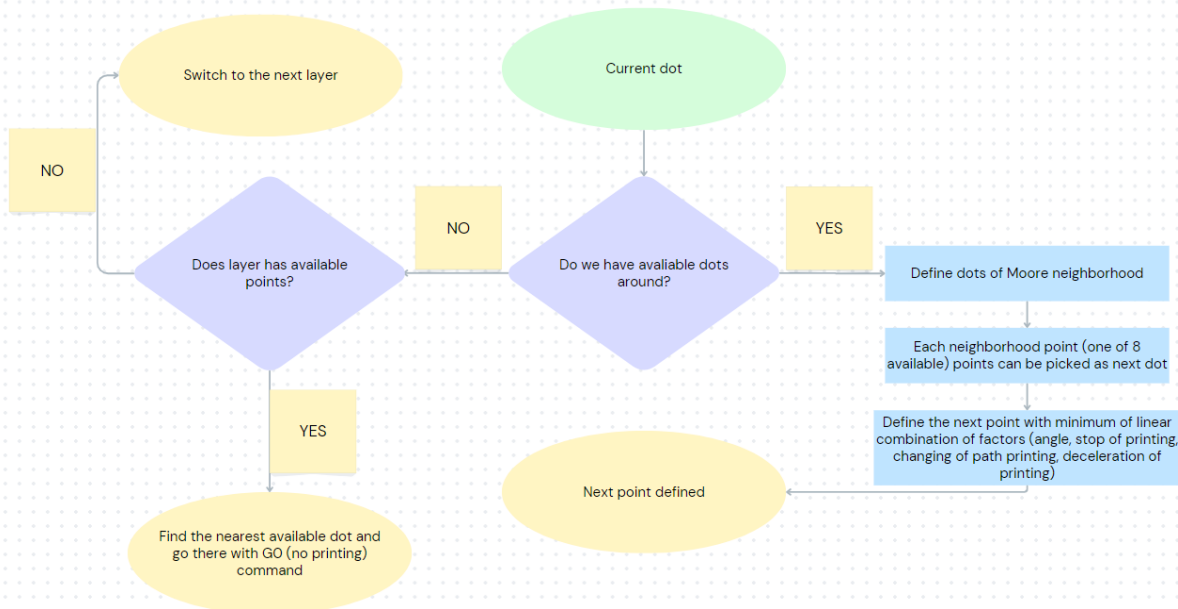


Figure 1 - Flowchart of the algorithm for finding the (k+1)-th point

### Algorithm of the program.

#### Selection of the starting point of printing:

In the implemented algorithm for optimizing the print path, the selection of the starting point on each layer is performed as follows: first, the point that is the center of the inscribed

circle covering the shape is selected. If the center of the circle belongs to a shape, this point is selected as the starting point. In the case when the center does not belong to the figure, a circle is built with a minimum radius in the center of the inscribed circle, "cutting" this circle from the original figure. The radius of such a circle gradually increases until there is an intersection with the figure. From the set of intersection points, the starting point is selected to build the print path. Usually, this point is chosen randomly to make the algorithm more adaptable and unique for different printing scenarios. This approach helps to ensure the efficiency and accuracy of determining the starting point to optimize the trajectory of printing on a 3D printer.

#### **Choosing the next point:**

In this algorithm for optimizing the print path, after determining the starting point on each layer, a complex process of selecting the next point for the transition is performed. In the course of studying the optimization algorithm for selecting the next point on the print layer, it was determined that the implementation of the Moore circle turned out to be more efficient compared to the Neumann circle. The Moore circumference, which takes into account coordinate axes and diagonals to define adjacent points, allows for more flexible and accurate navigation in space compared to the more constrained Neumann circumference, which takes into account only the major coordinate axes.

This selection is based on the fact that Moore's Oak provides a wider range of options for selecting points, which allows you to more effectively avoid areas with undesirable properties, such as slowing down, stopping or changing the direction of printing. This result highlights the importance of choosing the optimal circle to improve the speed and accuracy of the 3D printer printing process.

Each of the eight adjacent points is considered as a possible step, and for each point, a weight is calculated based on costs and undesirable actions such as stopping, slowing down, and changing the direction of printing.

This process of choosing the optimal route takes into account a variety of factors, each of which has its own weight factor, reflecting the impact on the final result. Such a weighing system allows for a deeper and more accurate assessment of the effectiveness of each possible step.

In addition, a certain point with minimal costs is chosen as the best option for further transition. This solution helps to maintain an optimal trajectory, simplifying and speeding up the 3D printing process. The costs and undesirable actions considered are taken into account to achieve maximum productivity and high quality of the manufactured parts.

Thus, the entire layer is represented by the optimal sequence of points for printing.

#### **Move to the next layer**

When switching to a new layer, the selection of the starting point cannot be based on the previously used algorithm, since it assumes the absence of previous points and minimal costs for moving the head. In the case of having information about the last point on the previous layer, it is reasonable to choose the starting point on the new layer according to minimizing the distance to this last point. This usually means choosing the projection of the last point of a layer onto a new print layer, but in cases where a point is not included in the outline of a

shape on a given layer, the process involves calculating the distance to each point (with a limited number of points selected from a small circle of the imaginary projection of the last point onto a new layer) and selecting a point with a minimum distance.

This process is repeated until the entire model is covered by a sequence of coordinates.

**Results of the program.** The result of this work is a software product that can be used to build an optimal trajectory for printing 3D shapes. The following is a flowchart if it describes how the program works.

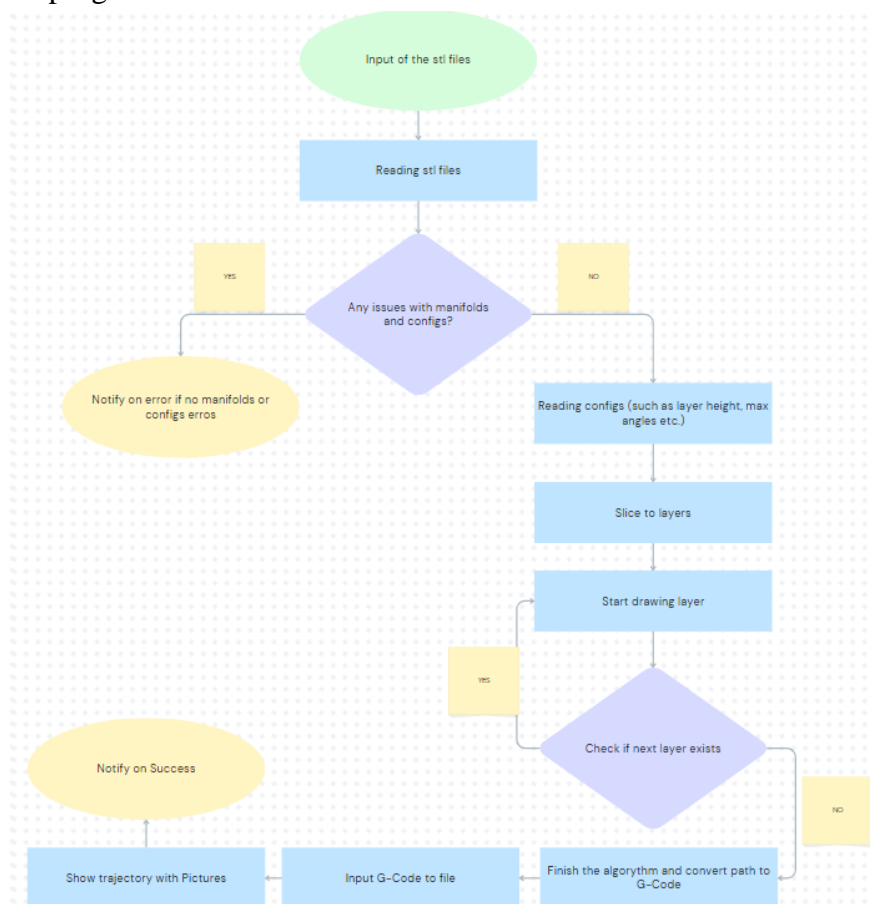


Figure 2 - Software Product Flowchart

Since the product was implemented using the Python language and the PyQt5 graphics library, a graphical interface was created for desktops and laptops with MacOS, Windows and Linux systems, the code was compiled using the Auto PY library to run this product conveniently, and an executable printer.exe was created to run this product.

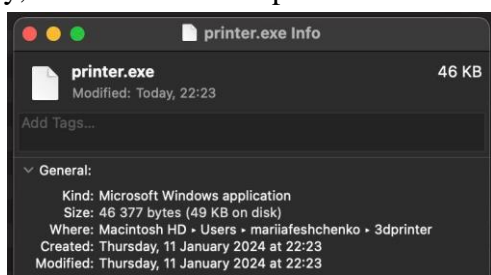


Figure 3 - Characteristics of the application executable file

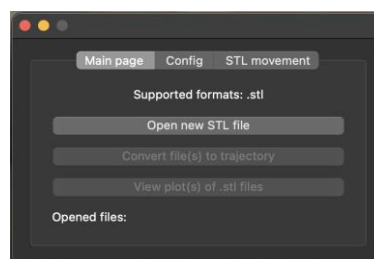


Figure 4 - The main page of the application

Immediately after the launch of the product, the user is greeted by the interface of the main menu, from which interaction with the product begins. Since the product was implemented for one type of printer, at the moment there is support for only one format of graphic objects - stl, one of the most common formats. Fig. 4 shows the main page of the product.

Since the necessary files have not yet been selected, there is no possibility for the user to interact with anything except the "Open new STL file" button, which allows you to open Explorer and select the necessary files to interact with the software product:

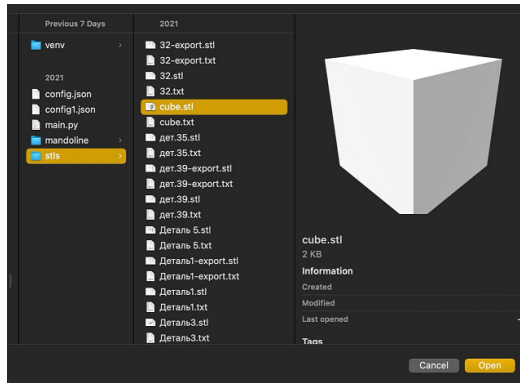


Figure 5 - Selecting a file to build a trajectory

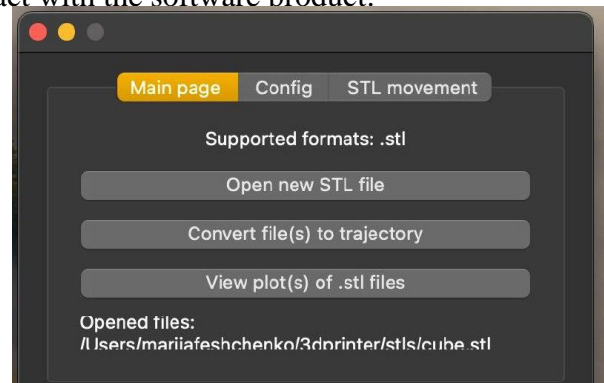


Figure 6 - Home screen after selecting a file

After selecting the required file, the user is given the opportunity to interact with other parts of the software in the future and information is displayed regarding which files the program is currently interacting with Figure 6.

Also, the user is given the opportunity to interact with other software tabs, namely: "Config" and "STL movement".

By opening the Config tab, the user can see that it is possible to individually select the print settings, since each 3D printer may have its own limitations and configurations. Among the most common that have been added to the product configuration are:

- Print track diameter (default value 1.3mm);
- Printing speed (8.1(6) mm/s);
- Largest angle between two layers (5).

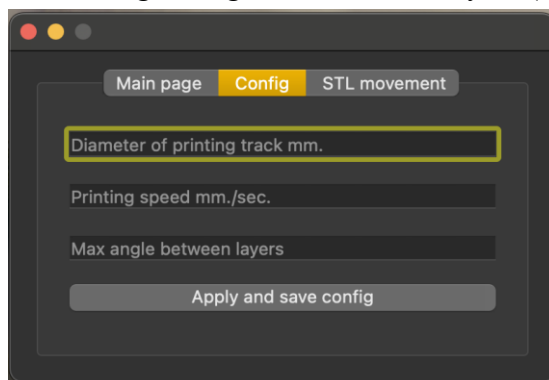


Figure 7 - Tab for adjusting individual printer parameters

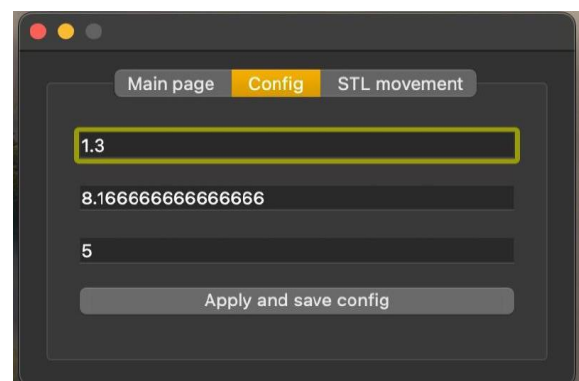


Figure 8 - Setting the default parameters of the printer

The default values in the software look like Figure 8.

Also, sometimes there are cases when the shape located in the .stl file will have its conditional center not at the origin or it needs to be transferred due to the specifics of the 3D printer, in which case it is possible to fix this using the third tab "STL movement", which will allow the user to center the shape as needed:

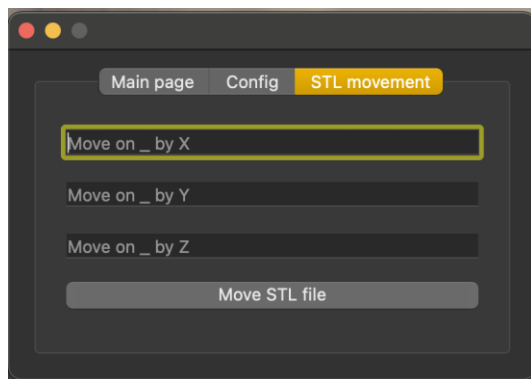


Figure 9 - Functionality for centering the shape to be printed

When you return to the "Main menu" tab and click the "Convert file(s) to trajectory" button, all selected stl files will be converted to path files with a .txt extension, and a print trajectory graph will appear at the end. Let's look at the example of two existing .stl models, one of which is a cube and the other is a ring. Fig. Figure 10 shows the print path for the cube figure, which was read from the cube.stl file, and Fig. 11 - the print path for the ring shape that was read from the round.stl file.

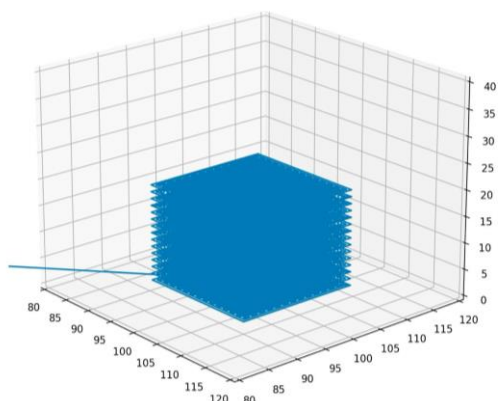


Figure 10 - Cube shape printing trajectory

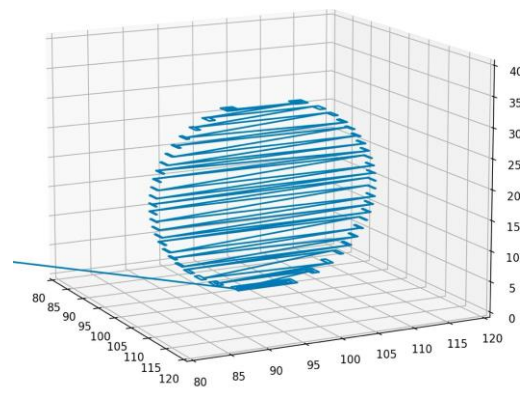


Figure 11 - Trajectory of printing a ring shape

Also, when you click on the "View plot(s) of .stl files" button, the user will be given a trajectory of building a figure in animated variations, which allows the user to observe the process of this construction from the beginning to the final strokes. Consider this using the example of Fig. 12-14 for a cube figure.

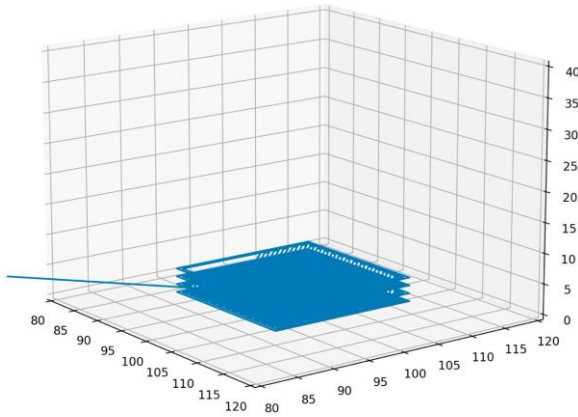


Figure 12 - Trajectory of printing a cube shape by 25 percent

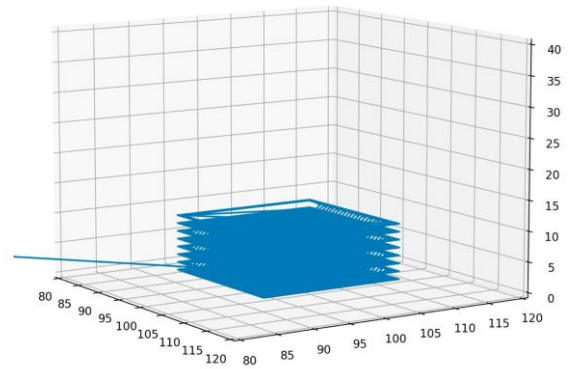


Figure 13 - Trajectory of printing a cube figure by 50 percent

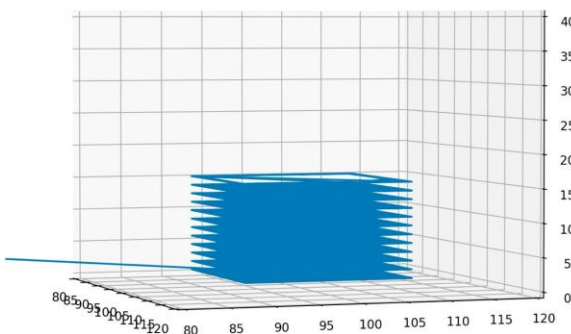


Figure 14 - 75 percent cube shape printing trajectory

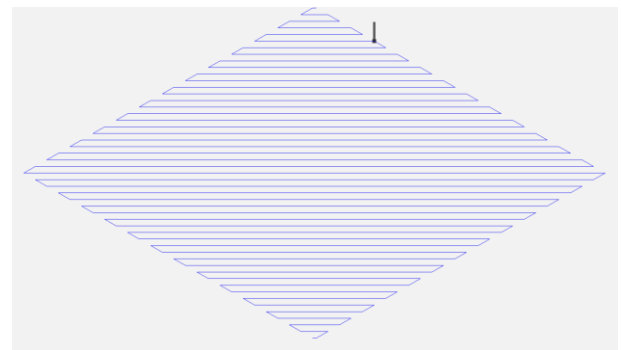


Figure 15 - Printing trajectory of the first layer of the cube shape

Since the user is given the opportunity to observe the progress of drawing the trajectory of the figure, it is possible to adjust the g-code at the necessary stages, if such a need arises.

The result of the program is g-code, which issues commands for a 3D printer, building each layer step by step using the g-code simulator, you can see the printing process and follow the implemented trajectory.

**Conclusions.** As a result of the study, a software product was developed aimed at building and implementing the optimal trajectory of printing on a 3D printer using a systematic approach. The importance of trajectory optimization to reduce printing time and improve the quality of manufactured products is substantiated.

During the implementation of the work, the skills of software development were improved, using modern technologies using the Python programming language, its libraries and algorithms. A step-by-step approach is made to maintain the system for various existing parts. Some algorithmic solutions have been mastered, the functional support of various systems has been improved, and the skills of working with 3D printers have been improved.

The developed software tool allows the user to import the object model in STL format and then automatically generates the optimal path for 3D printing. An important step is the

ability to visualize the figure at different stages of printing, which allows you to analyze and improve the manufacturing process.

The developed algorithm for building a trajectory is based on a systematic approach, which allows you to effectively take into account various parameters and constraints. It takes into account the path length spent printing each layer and uses advanced algorithms to solve the path optimization problem.

The final product is capable of displaying the G-code, which is the standard for numerical control of devices, and allows precise instructions to be transmitted to the printer to reproduce the selected model. Taking into account the above, the developed tool meets modern requirements and standards in the field of 3D printing, providing the user with convenience and high accuracy in the manufacture of objects. In the future, the resulting system can be improved and scaled by adding new features, and, if necessary, its use can be expanded.

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#### *Алгоритми та методи в динамічних задачах оптимального розміщення доріжок у тривимірному друку*

*Робота присвячена вивченню алгоритмів і методів розрахунку оптимальних траєкторій при 3-D друці в динамічній постановці задачі теорії оптимального розбиття множин. Актуальність цього завдання полягає в значній актуалізації 3-D друку, як при виготовленні медичної, військової, так і виробів подвійного призначення. У наш час технологічні рішення для 3-D друку дозволяють користувачам виготовляти деталі від дитячих іграшок із пластику до деталей ракет із високолегованої сталі. В умовах війни 3D-друк став для України інструментом удосконалення зброї, що дозволяє створювати експериментальні продукти в невеликих кількостях без ресурсомістких досліджень, досліджувати їх ефективність та застосовність на практиці, вдосконалювати та запускати в масове виробництво. У статті розглянуто математичні аспекти побудо-*



*ви траєкторій 3-D друку з урахуванням обмежень, що висуваються виробниками в математичній постановці. Такий підхід дозволяє отримати оптимальні рішення, мінімізувати час і витрати на доопрацювання деталей, і в цілому скоротити час і витрати на їх виготовлення. Слід зазначити, що такий підхід дуже актуальний в часі, що обумовлено великою кількістю нових винаходів, які розробляються в різних сферах людського існування. Приклади застосування технології показано на простих прикладах, які дозволяють оцінити роботу алгоритмів та методів на модельних тривимірних об'єктах. Більш складні тривимірні об'єкти можна, як правило, представити як сукупність простих, що підкреслює актуальність даних досліджень.*

*Ключові слова: математичне моделювання, теорія оптимального розбиття множин, динамічна задача, 3-D друк, оптимальна траєкторія.*

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