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METHOD OF SEQUENTIAL APPROXIMATION USING TO DETERMINE THE METAL-RESIN ANCHOR LONGITUDENAL FORCES DISTRIBUTION DURING ITS PULLOUT TESTING

Abstract. The results of SEPT investigating, as a rule, are general parameters as cohesion and shear strength and system rigid. The parameters achieved without taking into account fixed compound parameters and efforts parameter distribution were not analysing. Attempt to obtain the analytical form distribution at "anchor bar-fixing compound" and "fixing compound-rock" contacts has made. Present paper devoted to comparison the solving of generalized task of M.E. Zhukovskiy specification with taking into account the specialty of anchor rebar- fixing compound-rock system with the parameters obtained with finite element method (FEM). Obtained results demonstrate correspondence to viewpoints of metal resin anchor loading mechanism. Simplified formulas of efforts dependences in fixing compound shell vs construction system parameters were obtained. Do results comparison with ones, what obtained with FEM method. Graphic dependencies are presented. Conclusions about main parameters of efforts along resin anchor components distributions done. Conclusions on using effectiveness of proposed method for analysis SEPT testing parameters obtaining made.
Keywords: sequential approximation method, SEPT investigating, metal-resin anchor longitudenal forces distribution.

1. Introduction. Fastening of mining works with metal-polymer anchors (MA) has become widespread in Ukraine. A departmental program has been developed to involve mining enterprises in the use of MA for opens fastening in mining. However, this type of fastening also has its drawbacks. These, in the conditions of Ukraine, include: irregular provision of components for anchor fastening (AF), deviations from the design requirements for fastening technology. The irregular supply of components, and in particular ampoules with a fixing compound (FC), in turn leads to a level of adhesion to the surface of the anchor hole that does not correspond to the design. This situation leads to an unsatisfactory level of fastening of both the roof and the bottom of open in the working, which does not provide the required level of mixing of the FC, and accordingly cannot provide the design adhesion strength. The strength of the adhesion of the fixed MA of roof is a fundamental parameter that determines the effectiveness of fastening not only an individual anchor, but also the entire system as a whole [1-3]. This is especially important when intensification of mining operations [4-6], and in conditions of weak and water-logged rocks [7]. The stronger the adhesion, the shorter the working zone of the anchor and the longer the zone of full resistance, over which the anchor resists the movements of the roof with the greatest possible load of the anchor bar [8]. To check the strength of the adhesion of the rebar in the borehole, in practice, pulling out the fixed MA under its operating conditions is used. To check the properties of the FC, taking into account the irregularity of the provision of FC ampoules, the standard fixed anchor is pulled out. Taking into account the complexity of the processes of interaction of the FC with the surface of the borehole, existing mathematical models do not lead to the obtaining of analytical expressions for the parameters of the stress-strain state (SSS) in the system "anchor bar-FC-rock". The purpose of the research presented in the article is to obtain in analytical form the parameters of the SSS in the system "anchor bar-FC-rock", which will allow to carry out research on the parameters influence on the strength of rebar fixation in the borehole, and accordingly will allow to carry out an assessment of the sensitivity of the risk function to the parameters.

2. Methods. For an anchor with fixation by polymer FC, the anchoring effect is ensured by its adhesion to the rock, and the adhesion strength can easily exceed the strength of a steel anchor bar [9]. To eliminate such situations, the rebar (це є анкерна штанга, а анкер це є зафіксована анкерна штанга з допомогою фіксуючої суміші) is specially fixed with short-length ampoules of FC. This type of testing is called SEPT (Shot Encapsulated Pull Test) and is an internationally recognized method for measuring the anchoring efficiency or adhesion properties of fully filled anchors (FFA) of the roof. Standard SEPT equipment is presented in figure 1 [10].

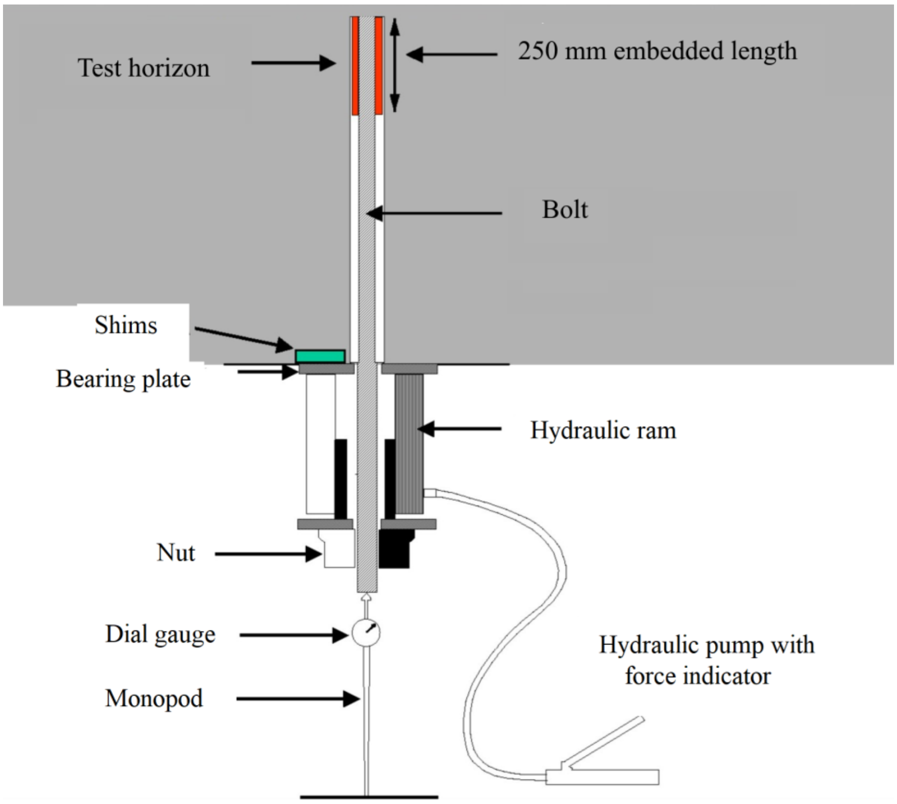

Figure 1 - Equipment for conducting SEPT tests

The load transfer capacity is a factor that determines the strength of the anchor and

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ensures the effectiveness of the fastening of the rock mass. In [11], its definition is given as the ratio of the change in load to the length of the section along which the anchor is fixed. In [12], this capacity is defined as the maximum value of the stresses acting on a unit area of the roof anchor surface. More effective fastening systems are characterized by a high load transfer capacity, i.e. with a high load at small displacements.

Two models that relate to the nature of load transfer in FFA have been operating for 30 years. One of them is associated with the transfer of nonlinear loads in pull-out tests performed in laboratory and natural conditions. An alternative model is used to transfer linear loads observed in field studies, but under the condition that the transfer of loads was initiated by the separation of the roof layer. It is believed [13] that these two methods differ only in the way the anchor bar is loaded. For the convenience of performing the axial tensile load test, the free end of the FFA is usually applied using a hydraulic jack. At the same time, the reaction of the jack causes a compressive load on the rocks surrounding the anchor, acting on the surface in the vicinity of the borehole. It is argued [14] that the most likely mechanism of loading of the roof anchor in real conditions is caused by delamination and movement of the rebar in opposite directions to the movement of the adjacent layers. It is argued that the analysis and interpretation of the results obtained in pullout tests must be carried out very carefully, since there is a tendency to overestimate the level of support compared to that which should be proposed for rock support by load transfer and restraint. Under SEPT testing, the adhesion factor (adhesionstrength), shear strength and stiffness of the system can be calculated by the formulas:

- adhesion strength [kN/mm]

$$GP = \frac{F}{l}; \tag{1}$$

- shear strength [MPa]

$$\tau = \frac{F}{\pi d l}; \tag{2}$$

- stiffness of the system [kN/mm]

$$K = \frac{\Delta F}{\Delta D}, \tag{3}$$

where F – force causing the anchor to move out of the borehole, kN; ΔF – changes in force magnitude (typically from 20 to 80 kN); ΔD – changes in the magnitude of deformations caused by changes in forces, mm (figure 2) [10]; l – length of the fixed section of the anchor (usually the working section is 250 mm); d - hole diameter, [mm].

The key to using these relationships is that the shear of the anchor bar (AB) must occur at the contact surface “FC - AB” or “FC - rock”. In weak roof rocks, the contact surface “FC - rock” controls the shear mechanism. If the rocks are stronger, the clutch failure may occur at the contact surface “FC - AB”. In the case where shear does not occur and the applied forces exceed the shear strength, equation (2) can be used to calculate the shear stresses for the applied forces, and the stiffness of the system can be calculated using formula (3) [10]. Satisfactory anchoring quality, at which the anchor capacity is equal to or significantly exceeds the yield strength of the AB material, was determined in SEPT testing by minimal movements (high bond stiffness).

Unsatisfactory anchoring quality is characterized by significant shear movements of the rebar and the pullout loads do not exceed the yield strength of the rebar material [15]. It is stated [16] that the load distribution during anchor pullout is determined by the ratio of the elastic modulus of the FC (ER) to the elastic modulus of the roof rocks (ERR). In the case when: ER/ERR > 10 - the load distribution is linear, and when ER/ERR < 10 - the load distribution is nonlinear.

The nature of the load distribution can now be determined using an instrument anchor (IA). An IA can be used to measure the loads on the anchor that arise during various mining operations in the mine. These anchor loads are compared with the yield loads [17], which allows determining the optimal design of the mine roof fastening system (anchor length and distance between them). Another way to design a roof-to-roof anchor system is to calculate the total stress (axial bending stress) for each measurement section [12]. If the total stress (along the measurement line) exceeds the maximum allowable stresses, then subsequent measurements can be made only after changing the parameters to reduce the stresses in the anchor, namely: reducing the anchor spacing, increasing the number of anchors in a row; or increasing the diameter of the anchor rods. But there are certain difficulties in this way.

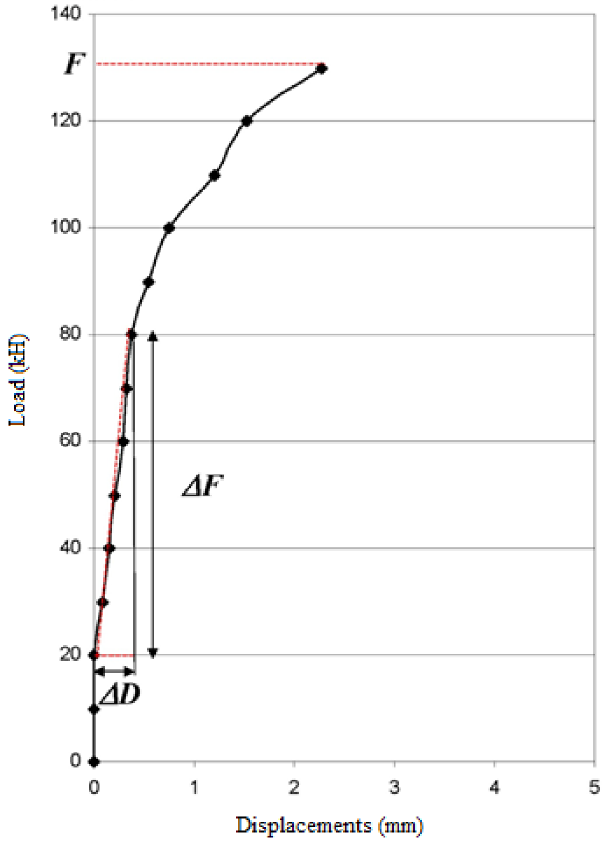

Figure 2 - Typical view of the results of pulling out short anchors

Although instrumental anchors can provide information about the distribution of axial and bending loads along it, their use has the following disadvantages [17]:

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- since the instrument anchor (IA) вже було позначене вище. IA has samples along its length, their presence can lead to incorrect determination of the cross-sectional area;
- the values of axial and bending loads can reach maximums in the intervals between measurement sections and thus will not be measured (recorded);
- centering of strain gauges is critical for obtaining reliable results;
- sensor failure may be the result of a break in the connecting conductors or excessive loads and may lead to incorrect reading of one or more axial load values [18].

Thus, an urgent scientific problem is to determine the distribution of forces along the anchor by analytical methods, for example, by solving the generalized problem of M.E. Zhukovsky.

In work [19], the solution of the classical problem of M.E. Zhukovsky on the distribution of tensile forces in a screw pair rock - screw (anchor bar) [20] was used to study the distribution of tensile forces. As the analysis of the results of the solution of the problem showed, the greatest load is experienced by several turns in the vicinity of the free surface within which the load forces were distributed. The absence of a fixing mixture layer loading in the considered scheme did not allow the full use of the obtained results to clarify the distribution of forces at the contact of the rock and the rezin-metal-anchor (RMA).

The purpose of the study presented in the article is to obtain in analytical form the parameters of the SSS in the system "AB-FC-rock", which will allow to carry out research on the influence of parameters on the strength of anchor rebar fixation in the borehole.

In connection with the above, to solve the problem of force distribution on the contact surfaces of this scheme, it is possible to apply with certainty the solution of the generalized problem of M.E. Zhukovsky [21] (figure 3). Using a modification of the generalized problem of M.E. Zhukovsky, the parameters of the SSS in the body of the FC shell [21,22] are obtained. The obtained solution, which is difficult to calculate, is reproduced in a more convenient form using the method of successive approximation (MSA), revealing the patterns of the influence of the initial parameters and in particular: the magnitude of the preload q [Pa], the diameters of the anchor bar and the borehole d_a, d_{vt}, [m], and their elastic moduli E_a, E_{vt} [Pa], the leads of thread h_a, h_{vt} [m], and the length of the rebar fixing section l [m], on the magnitude of the average integral value of the forces in the shell from the FC.

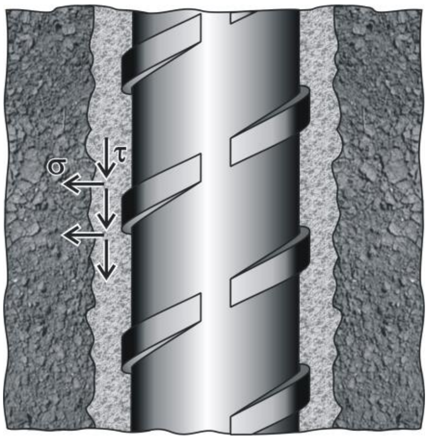

Figure 3 - Load diagram of RMA fixation in a borehole. Where σ is the compressive stress of the anchor bar in the borehole [Pa]; τ is the shear stress in the FC shell [Pa]

According to [21], the right rod reflects the body of the anchor rod, the middle one – the body of the sleeve, i.e. the shell of the FC, and the left rod – the massif of rocks (figure 4). The pitchs in which the rods touch, reflect the turns on the surface of the anchor rebar, the shell of the FC and the rock, respectively. The maximum number of turns of the thread is hereinafter denoted by the numbers n and n_1. The turns are numbered from bottom to top: (0), (1), ..., (n), and the corresponding fields between them: 1, 2, ..., n. The sought forces on the turns of the anchor bar and the sleeve are denoted as $p_0, p_1, ..., p_n$ [Pa], and on the turns of the FC shell and the rock – $t_0, t_1,.., t_n$ [Pa].. In the following, the anchor field will be understood as the part of the body located between two turns.

As in [14], for the forces in the k-th field of the anchor body we have the expression:

$$s_k^a = Q - \sum_{i=0}^{k-1} p_i \ . \tag{4}$$

Under the action of this force, the i-th anchor field increases its initial length by the amount of movement of the anchor rod $\delta_i^{\mathrm{a}} = \frac{s_i^{\mathrm{a}}}{E_{\mathrm{a}} F_{\mathrm{a}}}$, where $F_{\mathrm{a}} E_{\mathrm{a}}$,– the cross-sectional area of the anchor bar and the modulus of elasticity of the material, respectively.

The armature winding number i under the action of force p_i moves relative to its body upwards by an amount proportional to the force $f_k^{\mathrm{a}} = c_{\mathrm{a}} p_i$, where the proportionality coefficient c_a depends on the geometric dimensions and shape of the armature turns and is taken constant for all turns.

Deflection k-th turns of threads to downward view as:

$$\varDelta_k^{\mathrm{a}} = \delta_0^{\mathrm{a}} - \sum_{i=1}^{k} \delta_i^{\mathrm{a}} - f_k^{\mathrm{a}} = \delta_0^{\mathrm{a}} - \frac{h_{\mathrm{a}}}{E_{\mathrm{a}} F_{\mathrm{a}}} \sum_{i=1}^{k} s_i^{\mathrm{a}} - c_{\mathrm{a}} p_k , . \tag{5}$$

where $\varDelta_k^{\mathrm{a}}$ - downward displacement of the k-th anchor loop [m]; δ_0^{a} – displacement of the first turn in the positive direction of the axis along the anchor body downwards [m].

It should be noted that the directions to the left and right are counted from the FC shell. So to the right of the FC layer is the body of the anchor bar, to the left of it will be located the rock massif.

The output of the anchor number i under the action of the force t_i shift relative to its body upwards by an amount proportional to the force $f_k^{\mathrm{vt}} = c_{\mathrm{vt}} t_i$, where the proportionality coefficient c_{vt} depends on the geometric dimensions and shape of the armature turns and is assumed to be constant for all turns.

The downward displacement of the k-th right (inner) turn of the sleeve will be expressed by the formula:

$$\varDelta_k^{\mathrm{vt}} = \varDelta_0^{\mathrm{vt}} - \sum_{i=1}^{k} \delta_i^{\mathrm{vt}} + f_k^{\mathrm{vt}} = \varDelta_0^{\mathrm{vt}} - \frac{h_{\mathrm{vt}}}{E_{\mathrm{vt}} F_{\mathrm{vt}}} \sum_{i=1}^{k} \sigma_i^{\mathrm{vt}} + c_{\mathrm{vt}} p_k \ , \tag{6}$$

where $\varDelta_k^{\mathrm{vt}}$ - downward displacement of the kth right (inner) turn of the sleeve; $\varDelta_0^{\mathrm{vt}}$ - displacement (downward) of the lower surface of the sleeve; δ_i^{vt} - downward displacement of the i-th inner turn of the sleeve; $E_{\mathrm{vt}}, F_{\mathrm{vt}}$ - modulus of elasticity of the FC material and

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cross-sectional area of the FC respectively, c_{vt} - proportionality coefficient; σ_i^{vt} - force in the *i*-th sleeve field.

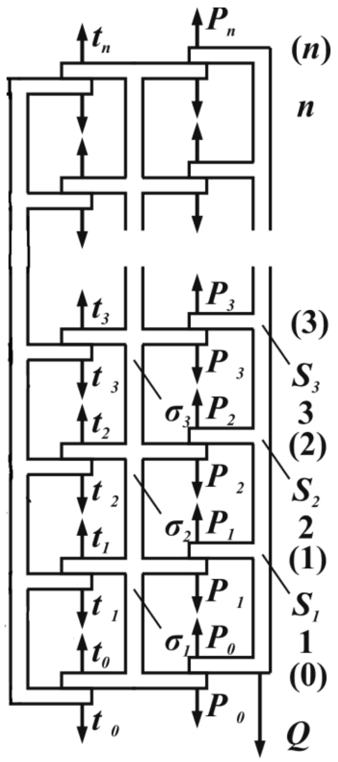

Figure 4 - Load diagram of RMA fixation in a borehole

From simple considerations of static equilibrium, it follows that

$$\sigma_k^{\text{vt}} = \sum_{i=0}^{k-1} (p_i^{\text{vt}} - t_i) = \sum_{i=0}^{k-1} (p_i - t_i), \tag{7}$$

where p_i^{vt} - forces from the anchor bar acting on the FC bushing; t_i – force on the *i*-th left (outer) turn of the sleeve.

From (5) and (6), observing that $\Delta_k^{\text{a}} = \Delta_k^{\text{vt}}$, we get:

$$\delta_0^{\text{a}} - \Delta_k^{\text{vt}} = \frac{h_{\text{a}}}{E_{\text{a}} F_{\text{a}}} \sum_{i=1}^{k} S_i^{\text{a}} - \frac{h_{\text{vt}}}{E_{\text{vt}} F_{\text{vt}}} \sum_{i=1}^{k} \sigma_i^{\text{vt}} + (c_{\text{a}} + c_{\text{vt}}) p_k \,. \tag{8}$$

After simple but cumbersome transformations, we obtain the expression for t_k and p_k:

$$t_k = \frac{\lambda_1 \lambda_3 Q}{8\operatorname{sh}\frac{\beta_1 + \beta_2}{2} \operatorname{sh}\frac{\beta_1 - \beta_2}{2}} \left[\frac{e^{-(k+1)\beta_2}}{\operatorname{sh}\frac{\beta_2}{2}} - \frac{e^{-\left(k+\frac{1}{2}\right)\beta_1}}{\operatorname{sh}\frac{\beta_1}{2}} \right] \tag{9}$$

$$p_k = \frac{\lambda_1 Q}{2\operatorname{sh}\frac{\beta_1 + \beta_2}{2} \operatorname{sh}\frac{\beta_1 - \beta_2}{2}} \left[\operatorname{sh}\frac{\beta_1}{2} e^{-(k+1)\beta_1} - \operatorname{sh}\frac{\beta_2}{2} e^{-(k+\frac{1}{2})\beta 2} \right] + t_k \,,$$

where

$$\lambda_1 = \frac{h_{\text{a}}}{E_{\text{a}} F_{\text{a}} (c_{\text{a}} + c_{\text{vt}})}; \qquad \lambda_2 = \frac{h_{\text{vt}}}{E_{\text{vt}} F_{\text{vt}} (c_{\text{a}} + c_{\text{vt}})};$$

$$\lambda_3 = \frac{h}{(c_{\text{vt}} + c_{\text{r}})}\left(\frac{1}{E_{\text{vt}}F_{\text{vt}}} - \frac{1}{E_{\text{r}}F_{\text{r}}}\right);$$

The flow of anchor number *i* under the action of force t_i will be displaced relative to his body upward by an amount proportional to the force $f_k^{\text{r}} = c_{\text{r}}t_i$, where the proportionality coefficient c_r depends on the geometric dimensions and shape of the turns on the surface of the borehole and is assumed to be constant for all turns.

$E_{\text{r}}F_{\text{r}}$ the elastic modulus of the rock environment and the cross-sectional area of the rock mass interacting with the FC material.

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_1\lambda_3 = 4\text{ch}\beta_1\text{ch}\beta_2 - 2(\text{ch}\beta_1 + \text{ch}\beta_2);$$

$$\text{ch}\beta_1 = 1 + \frac{\lambda_1 + \lambda_2 + \lambda_3}{4} + \sqrt{\left(1 + \frac{\lambda_1 + \lambda_2 + \lambda_3}{4}\right)^2 - \frac{\lambda_1\lambda_2}{4}};$$

$$\text{ch}\beta_2 = 1 + \frac{\lambda_1 + \lambda_2 + \lambda_3}{4} - \sqrt{\left(1 + \frac{\lambda_1 + \lambda_2 + \lambda_3}{4}\right)^2 - \frac{\lambda_1\lambda_2}{4}},$$

where h_{a}, h_{vt} - the leads of thread of the anchor bar and the FC shell, respectively.

The armature winding number *i* under the action of force p_i will shift relative to its body upwards by an amount proportional to the force $f_k^{\text{a}} = c_{\text{a}}p_i$, where the proportionality coefficient c_{a} depends on the geometric dimensions and shape of the turns on the surface of the anchor hole and is assumed to be constant for all turns.

In a first approximation, to determine the coefficients c_{a}, c_{vt} , you can use [22], where the authors proposed the following formula:

$$c_i = \frac{h_i}{E_iF_i} \quad ,$$

where *i* – number of the contact surface, starting from the surface of the anchor bar. As the proposed formula shows, only tensile deformations are taken into account. Since the material of the FC is a high-molecular substance, it is advisable to take into account bending, shearing and viscoelastic deformations. According to the calculation scheme of the problem of M.S. Zhukovsky, the FC layer is considered as a rod.

Let us consider a flat bending of a rod. If the length of the rod is co-dimensional with its height, then it is necessary to take into account the influence of shear deformations on its deflection [15]. In technical theory, the shear angle of the cross-section has the form:

$$\gamma = k(z)\frac{Q_{\text{y}}(z)}{GF(z)}$$

$Q_{\text{y}}(z)$ and P - shear force in section z [N]; $F(z)$ - cross-sectional area of the rod [m^2]; G – shear modulus of the rod material [Pa]; $k(z)$ – dimensionless coefficient (shear coefficient), which depends on the shape of the cross-section of the rod.

Guided by the averaging of shear energy, approximately the following is taken:

$$k = \frac{F}{I_{\text{x}}^2}\int_F \frac{S_{\text{f}}^2}{b^2}\,\text{d}F;$$

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where $S_{\mathrm{f}}(y)=\int\limits_{F} y\mathrm{d}F$ - static moment about the OX axis of the cut-off part of the cross-section. For a rectangular cross-section of the rod, we have:

$$S_{\mathrm{f}}(y)=\frac{b}{2}\left(\frac{h^2}{2}-y^2\right);\;\; I_{\mathrm{x}}=\frac{bh^3}{12};$$

$$\tau_{\mathrm{y}}=\frac{6Q_{\mathrm{y}}}{bh^3}\left(\frac{h^2}{4}-y^2\right);\; \tau_{\max}=\frac{3Q_{\mathrm{y}}}{2bh}=\frac{3Q_{\mathrm{y}}}{2F}.$$

For a rod of rectangular cross-section, the influence of the change in cross-section along its length on the value of the coefficient is neglected, therefore, we can take k=6/5. Since the bending and cross-section of the rod are related, we give the formulas for the additional deflection of the rod axis from the shear angle and from the action of bending forces:

$$\frac{\mathrm{d}y_{3\mathrm{c}}(z)}{\mathrm{dz}}=-k\frac{Q_{\mathrm{y}}(z)}{GF(z)}$$

The minus sign in the formula depends on the choice of direction of the shear force.

For bending, the relationship between deflection is as follows:

$$\frac{\mathrm{d}^2 y_{\mathrm{M}}(z)}{\mathrm{dz}^2}=\frac{M_{\mathrm{x}}(z)}{EI_{\mathrm{x}}(z)}$$

The differential equation of the elastic line of the rod, taking into account the influence of shear deformation, will have the form:

$$\frac{\mathrm{d}^2 y\;(z)}{\mathrm{dz}^2}=\frac{M_{\mathrm{x}}(z)}{EI_{\mathrm{x}}(z)}-k\frac{\mathrm{d}}{\mathrm{dz}}\left(\frac{Q_{\mathrm{y}}(z)}{GF(z)}\right),$$

where $y(z)=y_{\mathrm{m}}(z)+y_{\mathrm{sh}}(z)$ - total deflection of the rod (displacement of the center of gravity of the cross section along the *OZ* axis. In our case, we will have:

Bending moment and shear force $M(z)=-P(l-z);\; Q_y(z)=P$.

The boundary conditions for this case will be:

$$y(0)=0;\;\; \frac{\mathrm{d}y(0)}{\mathrm{dz}}=-k\frac{Q_{\mathrm{y}}(0)}{GF(0)}$$

Then the deflection of the rod axis takes the form:

$$y(z)=-k\frac{Pz}{GF}-\frac{P}{EI_{\mathrm{x}}}\left(\frac{lz^2}{2}-\frac{z^3}{6}\right)$$

The largest deflection is found under the condition $z = l$:

$$y(l)=-\left(k\frac{Pl}{GF}+\frac{Pl^3}{3EI_{\mathrm{x}}}\right)=\frac{Pl^3}{3EI_{\mathrm{x}}}(1+\lambda)$$

where

$$\lambda=\frac{k(1+v)h^2}{2l^2}$$

As shown by the calculations, the deflections are insignificant. Given this and the fact that the FC consists of a high-molecular substance, we will try to take into account the deformations caused by the viscoelastic components of the deformation. So, for the Kelvin-

Voigt model, we have:

$$\varepsilon = \varepsilon_1 = \varepsilon_2 ; \sigma = \sigma_1 + \sigma_2$$

Considering that $\sigma_1 = E\varepsilon_1 ; \sigma_2 = +\eta\varepsilon_2^*$, where E – elasticity modulus; η- dynamic viscosity the equality holds and ε' is the viscoelastic deformation components derevative:

$$\sigma = \sigma_1 + \sigma_2 = E\varepsilon + \eta\varepsilon_2^*$$

The equations can be applied both for shear stresses (shear deformation) and for tension-compression. After integrating the equation, we obtain:

$$\varepsilon(t) = \frac{\sigma_0}{E}(1 - e^{-2t}) \quad (10)$$

where t – time; $\lambda = E/\eta$ – the intensity of relaxation, and when $\sigma' = 0$, $\sigma = \sigma_0$ and σ' is the viscoelastic stress components derevative:

For our case we have:

$$\varepsilon(t) = \frac{l_0 - l}{l_0}$$

Thus, considering the fact that $\lambda = E/\eta$, we have an expression for the largest deflection under the conditions $z=l$

$$\delta_i = \frac{l^3 q_0}{3EI_x}(1+\lambda), \quad (11)$$

where $\lambda = \frac{k(1+\nu)}{2}\frac{h^2}{l^2}$ or for a rectangular cross-section of the rod λ=0,78; k=6/5.

In formula (1) where $\lambda^* = E/\eta$, and $\sigma_0 = q_0 / f_{vt}$, q_0 – axial load on the anchor rod. Then, taking into account these notations, we have:

$$\delta_i^* = \Delta l = \frac{q_0 l_0}{f_{vt} E}\left(1 - e^{-\lambda^* t}\right) \quad (12)$$

In the following, λ^* will be understood as $\lambda^* = E/\eta$.

Ultimately, the sum of the displacements of shear bending and viscoelastic deformation takes the form:

$$\delta = \delta_i + \delta_i^* = \left(\frac{l^3}{3EI_x}(1+\lambda) + \frac{l_0}{f_{vt}E}(1 - e^{-2^* t})\right) p_0 \quad (13)$$

Considering the fact that in the initial expressions for the displacement of the bushing protrusions, they were searched for in the form

$$\delta_{vt} = c_{vt} p_a ,$$

and taking into account (9) we will have

$$c_{vt} == \left(\frac{h_{vt}}{E_{vt}F_{vt}} + \frac{l^3}{3EI_x}(1+\lambda) + \frac{l_0}{f_{vt}E_{vt}}\left(1 - e^{-\lambda^* t}\right) + \frac{l}{E_{vt}F_{vt}}\right) \quad (14)$$

Thus, formula (14) takes into account all components of possible displacements, namely from bending, shear, viscoelastic deformations and compression.

Taking into account the special importance of the FC layer in the mechanism of load transfer from the anchor rod to the rock massif, we will consider as a force parameter the

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magnitude of the forces arising in the FC shell. (figures 3 and 4.). The loads on the contact surfaces "AB - FC" - pk and "FC - rock" - t_k are determined according to formulas (8), (9), and the forces arising in the FC shell, according to formula (7), namely:

$$\sigma_{\text{vt}} = \sum_{i=1}^{k-1} (p_i - t_i),$$

where p_i – forces acting on the contact surface of the "AB - FC" system; t_i – forces on the contact surface of the system "FC - hole surface".

The complex parameters characterizing the load in the system "AB - FC - rock" are the values of the average integral values of the quantities p_k; t_k; u_k; σ_{vt}, namely:

$$p_{\text{a}} = \frac{1}{L}\int_0^L p_k(\xi)\mathrm{d}\xi$$

$$t_{\text{vt}} = \frac{1}{L}\int_0^L t_k(\xi)\mathrm{d}\xi$$

$$u_{\text{avt}} = \frac{1}{L}\int_0^L u_{\text{avt}}(\xi)\mathrm{d}\xi \qquad (15)$$

$$\sigma_{\text{vt}} = \frac{1}{L}\int_0^L \sigma_{\text{vt}}(\xi)\mathrm{d}\xi .$$

where L - length of the anchor rod fixation section in the hole (L = 0.25 m); p_{a} – average integral value of forces along the anchor; t_{vt} – average integral value of forces along the borehole surface; u_{avt} – average integral value of anchor rod displacements; σ_{vt} – average integral value of forces in the body of the FC shell.

Let us try to further identify the patterns of influence of the initial parameters and in particular: the magnitude of the preload q, the diameters of the anchor rebar and the borehole d_{a}, d_{vt}, their elastic moduli E_{a}, E_{vt}, leads of periodic performances h_{a}, h_{vt} by the value of their average integral values in the system "AB - FC - rock. To solve the problem, we will use the developed MSA method.

To verify the reliability of the results obtained, we will compare the forces along the anchor rod p_k formula (9) with the results obtained using the finite element method (FEM).

In the FEM model of the anchor fixed in the rock, it is necessary to take into account the possibility of changing the strength of the fastening. For this, it is necessary to introduce special finite elements into the general mesh that simulate the contact between the rod finite elements approximating the anchor and the finite elements of the rock mass. Such contact can be described by a finite element of the usual form (as a finite element approximating the rock mass), but with the deformation and strength properties of the polymer. If we consider that the gap between the anchor rod and the wall of the hole is 3-5 mm, then to approximate the fastener, it is necessary to thicken the mesh of finite elements to reduce their size to the same value. This leads to a significant increase in their number and, accordingly, to an excessive increase in the requirements for computational and time resources for solving the problem. If we do not thicken the mesh, but use finite elements of the same size as for the rock massif, this leads to an artificial increase in the thickness of the polymer fastener, which distorts the obtained solution results.

To simulate the polymer anchoring of the anchor, we will use a special contact finite element.

When stretching, the stresses in the polymer fastener are related to the deformation by a linear dependence, and are limited by the value of the tensile strength T_p. Then, with a further increase in deformation, the material ruptures, and its tensile strength becomes equal to 0. After that, there is no resistance to stretching in the contact element. When compressed perpendicular to the contact element, the normal stresses are linearly dependent on the compressive strains. After the normal stress reaches the compressive limit value, the modulus of the relationship between normal stresses and strains becomes equal to the modulus of the surrounding rock massif in which the anchor rod is fixed. Thus, the relationship between normal stresses and normal strains is expressed by the relationship:

$$\sigma = \begin{cases} 0, & \text{at} \quad \varepsilon < \varepsilon_{\text{p}} \\ k_{\eta}\varepsilon, & \text{at} \quad \varepsilon_{\text{p}} < \varepsilon < \varepsilon_{\text{c}} \\ K_{\text{M}}\varepsilon, & \text{at} \quad \varepsilon_{\text{c}} < \varepsilon \end{cases},$$

where k_{η} - normal stiffness of the fastener; K_{M} - rigidity of the rock massif and ε_p – tensile deformation; ε_c – compress deformation.

Figure 5 presents a comparison of the graphical dependences of the contact forces along the anchor rod p_k based on the solution of the generalized problem of M.E. Zhukovsky (9) and the forces obtained using the FEM model. The results were obtained with the following initial data: L_a= 2.4 m; L_{fix}= 0.25 m; d_a = 22 mm; d_{vt} = 28 mm; E_a = 2.1 10^{11} Pa; E_{vt} = 0.9 10^9 Pa.

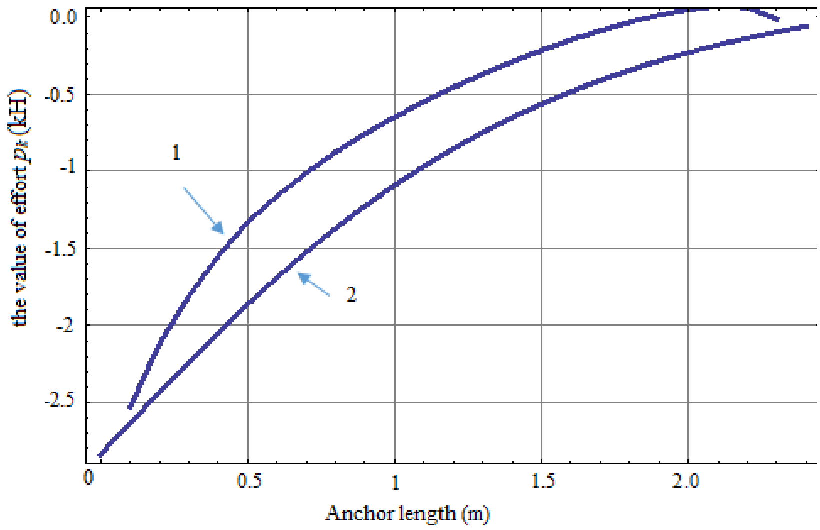

Figure 5 - Graphs of force distribution along the anchor rod: 1- according to the calculation of the FEM method, 2- according to the proposed in (9)

3. Results and discussion. Good correspondence of the force distribution graphs along the anchor rod allows us to be confident that all other dependencies of the force and displacement distributions will also correspond to reality. In the future, it is planned to conduct research in this direction.

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After performing the procedures of the above algorithm, the dependence of the functions p_k; t_k; u_k; σ_{vt} from the parameters q, d_a, d_{vt}, E_a, E_v, h_a, h_{vt} take shape:

$$p_k = A_p \frac{d_{vt}^{3.73} E_{vt}^{3.7329} h_a^{0.966} h_{vt}^{0.03417} q}{d_a^{3.91} E_a^{0.734894}};$$

$$t_k = A_t \frac{d_{vt}^{3.5197} E_{vt}^{4.221376} h_a^{0.69105} h_{vt}^{0.307915} q}{d_a^{3.6932} E_a^{0.691915}};$$

$$u_k = A_u \frac{d_{vt}^{0.00095825} h_a^{2.7050582} q}{d_a^2 E_a^{1.00018} E_{vt}^{0.00102} h_{vt}^{0.002054}}; \qquad (18)$$

$$\sigma_{vt} = A_\sigma \frac{d_{vt}^{3.53} E_{vt}^{2.35} h_a^{3.68} q}{d_a^{3.706} E_a^{0.6942} h_{vt}^{1.77}}$$

where A_p=8.26237 10^{-14}; A_t=4.47933 10^{-36}; A_u=41619; $A_{\sigma vt}$=7.98866 10^{-79}.

The obtained formulas (18) allow us to calculate the average integral values of the force distribution in the system “AB – FC – rock”. It should be noted that the movement of the end of the anchor consists of two terms, namely the movement in the locking part l_{lock} of the anchor and the movement of its unfixed section l_{free}:

$$U_0 = A_u \frac{d_{vt}^{0.00095825} h_a^{2.7050582} q}{d_a^2 E_a^{1.00018} E_{vt}^{0.00102} h_{vt}^{0.002054}} + \frac{Pl_{free}}{E_a F_a}$$

4. Conclusions.

1. For the first time, the parameters of the SSS in the system "AB-FC-rock" were obtained in analytical form, which will allow studying the influence of parameters on the strength of anchor rebar fixation in the hole;

2. Analysis of the relative errors of the obtained representation (22) showed that the error of the representation in comparison with the formulas of the obtained solution (16) did not exceed 7%.; comparison of the distribution of forces along the length of the fixed section by the developed and FEM method showed qualitative agreement. The relative error did not exceed 10%, which is acceptable for engineering applications.

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Використання методу послідовної апроксимації до визначення повздовжніх зусиль в металополімерному анкері під час проведення тестів на висмикування

Стаття присвячена порівнянню результатів досліджень на висмикування металополімерних анкерів з врахуванням особливостей в системах анкерна штанга-фіксуюча суміш-гірська порода з параметрами отриманими з використанням методу скінчених елементів. Отримані результати демонструють відповідність механізму навантаження металополімерних анкерів. Отримані спрощені формули для визначення залежностей у оболонці фіксуючої суміші від конструктивних параметрів системи. Виконано порівняння отриманих результатів з параметрами отриманими методом скінчених елементів. Представлено графічні залежності. Зроблено висновки про розподіл основних зусиль вздовж металополімерного анкеру. Зроблено висновок про ефективність застосування запропонованого методу до аналізу результатів SEPT тестування.

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