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Oleksandr Kuzenkov

ALGORITHMS AND METHODS IN DYNAMIC PROBLEMS OF OPTIMAL PLACEMENT OF FIRE GROUPS

Abstract. *The work is devoted to the urgent task of deploying fire groups of air defense forces to cover the lines of air attacks. To solve the problem, modern methods of the theory of optimal set partitioning, mathematical modeling using differential equations and their systems, a specialized software package has been developed, which includes a mobile and browser application, is used. When developing the software package, modern programming languages and technologies were used, an overview and comparison with existing solutions in different countries was made. A retrospective analysis was carried out, criteria for the optimality of the solution being sought were developed, advantages and disadvantages of the approach were determined.*

The use of methods of the theory of optimal set partitioning allows you to analytically determine the criteria for the quality of the solution, conduct an analytical study of admissible solutions and determine the optimal one, and approbation of analytical results in practice and examples with practical input data increases the accuracy and relevance of the results obtained analytically.

Nowadays, the issue of dynamic deployment of fire groups is very relevant for Ukraine. The decision to create and operate fire groups was made and implemented only from the beginning of the full-scale invasion of the aggressor country and became widely used during the massive use of attack drones by the aggressor country against civilian, military and energy facilities of Ukraine. The effectiveness of the use of fire groups has been proven in practice, but the method of their placement, as a rule, is heuristic, that is, not optimal.

Keywords: *optimal partitioning of sets, dynamic problem, fire group, air defense, software complex.*

Entry

Today, the issue of air defense for Ukraine is a matter of the existence of not only the energy system, but also the safety of civilians and the military. Numerous

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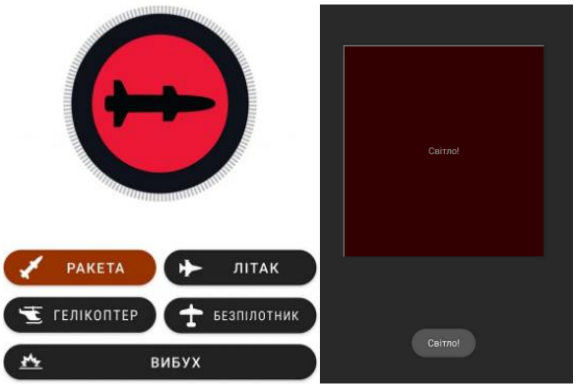
air raid sirens, which are heard daily in different cities of Ukraine, have unfortunately become commonplace, and only the professional work of air defense servicemen and the aviation command allows you to fight numerous air targets that fly into Ukraine. It should be noted that for almost three years of war, the nature and means of air attack in Ukraine have undergone significant and fundamental changes. Thus, in 2022, attacks with X-101/555 cruise missiles were characteristic. At the end of 2022 and throughout 2023, ballistic missiles, Shahed 136 attack drones and Orlan-10 reconnaissance aircraft were added to the list of air targets. At the end of 2023 and throughout 2024, it was possible to observe a decrease in the number of missile attacks, but a significant increase in the shelling of the territory of Ukraine by attack drones and the use of reconnaissance drones for ballistic strikes by the enemy, on strategically important military and civilian targets. This situation required a prompt response and adaptation of means of detecting and destroying air targets. It should be noted that along with the adaptation of detection means, the means of destruction have also undergone significant changes. In particular, this is due to the lower cost of Shahed-136 attack UAVs compared to surface-to-air missiles typical for the Armed Forces of Ukraine. It is for these reasons that mobile fire groups were used to destroy air targets from small arms of various calibers, as well as detection means other than typical radars.

Subject statement of the problem

Due to the limitations in the use of standard means of detecting air targets and the low altitude of their route, it was decided to develop mobile applications with the help of which, including the civilian population, could promptly notify the air defense forces about the flight of an enemy aircraft, missile, drone, etc. Taking into account the fact that the efficiency of informing in this matter plays an important role, the interface of the program for users is simplified as much as possible (Fig. 1-b). Fig. 1. Here are some examples of software applications used for notifications.



a) b)

Figure 1 - Graphical interface of warning systems
(a – EPPO appendix, b – alternative notification system "Light").

Figure 1(a) shows the software developed by Odesa programmers, which is widely used on the territory of Ukraine and is designed to inform air defense forces about enemy air targets and their location. The peculiarity of this application is that the user can specify the type of air target if he is informed, and in new versions of the application, specify "UFO" if the type of air target could not be established.

Fig. 1(b) shows the alternative notification system "Light", which was developed and operated in the Dnipropetrovsk region from March 2022 to January 2023 (before the introduction of the EPPO annex throughout Ukraine) and is partially used today. The simplicity and confidentiality of the interface of the alternative warning system was explained by its use at the very beginning of the full-scale invasion, because the owners of this application sometimes had to overcome enemy checkpoints, and their phones could be searched.

As a result of the practical application of the alternative warning system "Light", a significant amount of information was obtained, which was used as a training sample for further improvement of the software package. It should be noted that at the stage when quantitative missile attacks were replaced by numerous attacks by attack drones, and the means of destroying air targets were replenished with mobile fire groups, a number of problems objectively arose that required thorough mathematical, as well as in some cases optimal solutions.

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Mathematical statement of the problem

For a rectangular area Ω with perimeter $P = 2 \cdot a \cdot b$ (where a - Width of a rectangular area, b - its length) along which an indefinite and unlimited number of enemy air targets can move by air, i.e. without reference to roads, it is necessary to find the following N points

$$(x_i, y_i), i = \overline{1, N}, \tag{1}$$

for which

$$\sum_{i=1}^{\infty} \mathrm{T}_i \cdot f(z_i, p_i) \to \max, \tag{2}$$

where

$$f(z_i, p_i) = \begin{cases} 1, if \ (z_i, p_i) \in \bigcup_{k=1}^{N} g_k(x_k, y_k) \\ 0, \ \ else \end{cases}, \tag{3}$$

$g_k(x_k, y_k)$ - circle centered at a point (x_k, y_k) and radius R,

$$T_i = \frac{t_i}{t_{cur} - t_i}, \tag{4}$$

t_i - date/time of the first crossing (entry) i-th enemy air target in the region Ω,

t_{cur} - current date/time,

N - Total number of mobile fire groups,

R - range of mobile fire groups.

The need to reduce the problem to the optimization problem is due to the fact that in most cases $\bigcup_{k=1}^{N} g_k(x_k, y_k)$ do not cover the entire territory of the specified region and even its perimeter.

Overview of existing solutions

Nowadays, there are many programs designed to notify civilians and the military about emergencies, but the features of each of them are focused on a specific place of application, and in the case of military use, focused on the peculiarities of war. Among the best examples of similar systems, the following should be highlighted:

1. WireX Alert System is an integrated alert system designed to address the diverse needs of users. The system supports various forms of notification, including SMS, email, and instant messaging. It is characterized by flexibility in setting priorities and configuring communication channels. The advantages include flexible configuration of alert channels and integration with various sources of information. Among the disadvantages is the limited set of built-in message templates.

2. AlertHub Pro is an operational alert system defined by its ability to combine alerts from different sources into a single interface. It is an integrated solution that allows users to receive and manage messages in real-time, as well as interact with other users through comments and discussions.

3. SafeAlert Plus is a security and alert system specially designed for organizations and enterprises. It provides extensive options for customizing user groups, access levels and events to be notified. The system can integrate with third-party security systems. The advantages include a high degree of customization and flexibility in configuration, as well as the ability to integrate with various security systems. Among the disadvantages, high implementation and support costs should be highlighted.

4. EverSafe Notifications is an alert system that stands out for its ability to provide reliable alerts through a variety of channels, including voice calls, instant messaging, and social media. The system is specifically targeted to the needs of groups with different needs, such as medical institutions and educational institutions. The advantages include a wide range of possible notification channels and adaptability for different types of users. Among the disadvantages is the rather limited function of personalizing messages.

5. NotiSync Enterprise Alerting is an alert system that uses intelligent algorithms to detect and notify about events in real time. The system provides

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automatic signal processing and a high level of adaptation to changing conditions. This makes NotiSync an effective solution for large enterprises and dynamic environments. Among its advantages are intelligent event detection algorithms and automatic adaptation to changing conditions. Among the disadvantages is the high cost of implementation and support.

6. NotifyMe360 is an alert system that focuses on the user experience. It allows users to confirm receipt of messages, send feedback, and quickly respond to crisis situations. The advantages include the presence of a mechanism for acknowledging the receipt of messages and real-time interaction with the user. A separate disadvantage is the limited ability to integrate with other systems.

7. Alertify Pro Suite is a comprehensive solution that combines not only an alert system, but also analysis and reporting tools. The system allows you to create dynamic alerts using various media formats and provides advanced tools for analyzing the effectiveness of campaigns. Among the advantages, it is necessary to highlight the wide possibilities for integration with analysis and reporting tools, as well as flexibility in creating dynamic alerts, and among the disadvantages is the high complexity of configuration for beginners.

8. SafeGuard Alert Management is an alert system that contains an advanced set of tools for managing alerts in crisis situations. The system includes automated processes to respond to different types of events and facilitates the rapid deployment of campaigns in emergency situations. Among the advantages are the implementation of automated processes for responding to various types of events and ensuring effective crisis management. Among the disadvantages are limited options for individual settings.

9. The RapidNotify Emergency Alert System is an emergency alert system specifically designed to serve large groups of people. It is distinguished by a high speed of sending messages and the ability to flexibly configure audience selection criteria. Among the advantages are the high speed of sending messages and the ability to flexibly customize the audience. Among the disadvantages is the unreasonably high cost of maintenance for small organizations.

10. OnSolve MIR3 is an integrated solution for crisis management and real-time alerting. The system includes a wide range of emergency response tools,

including integration with security systems. Among the advantages are the availability of integrated crisis management tools and the support of integration with various security systems. Among the shortcomings, it is necessary to highlight the high qualification of personnel for setting up and effective use of the system.

In the course of reviewing various public address systems, it can be determined that there are a significant number of solutions aimed at ensuring effective information of users in a wide variety of scenarios. Each of the defined systems has its own characteristics and advantages, which are important to consider when choosing a solution for a particular case.

The mathematical solution of the problem will be carried out with the involvement of the mathematical apparatus of the theory of optimal set partitioning [1]-[2], mathematical modeling using differential equations and their systems [3]-[4], a number of numerical methods and retrospective analysis [5].

Choosing a platform for implementation

Nowadays, the implementation of warning systems can be implemented, as a rule, on the basis of mobile devices (smartphones, tablets) or in web applications.

Development of a software package for mobile platforms

• designed for specific operating systems such as Android or iOS;

• designed specifically for mobile devices, their extensions and operating and file system specifications;

• can work offline;

• high-speed in working out compared to web applications;

• require permits for posting on trading platforms;

• the cost of development is higher than that of web applications;

• are developed using programming languages such as React Native, Dart, Swift, Kotlin.

The development of a software package in the form of a web application also has its advantages and disadvantages, namely:

• designed for browser access, but can also be used by mobile devices

• do not have access to hardware specifications

• require constant access to the Internet

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• slower than mobile apps
• does not require permissions for distribution
• the cost of development is lower than that of mobile applications;
• are developed using programming languages such as: Python, JS, TS, PHP.

Implementation of the software application

The developed software package includes:

● Database
● server part
● client part (mobile application)
● client part (web application)

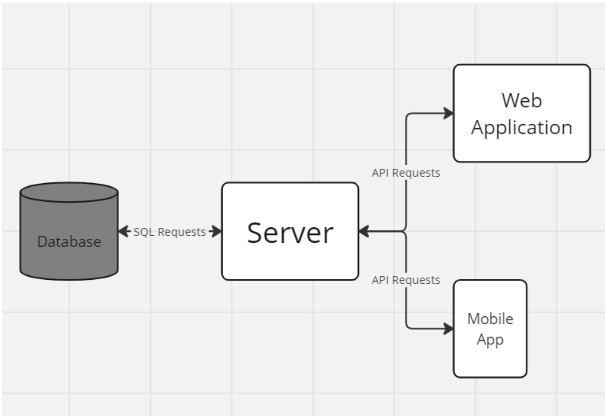

Figure 2 - Structure of the software package

The database was created on top of PostgreSQL, which made it possible to quickly and efficiently create tables and relationships between them. Fig. 2 shows the general structure of the software package, and Fig. 3 shows the structure of the database and the relationships between the tables:

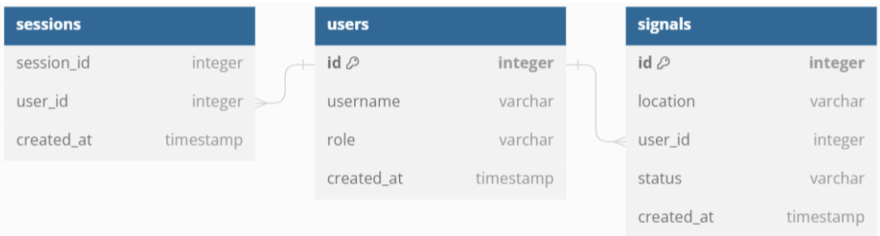

Figure 3 - Database diagram of the software package

This database schema has three main tables, one of which is users, having two roles: operator and user. The signals table stores information about the locations recorded by users and the time of receipt of the message. The session table allows you to keep track of when and which user logged in.

The server part was implemented using Nest.js technology, which is an add-on on top of the Node.js library. The main functional integrations of the backend are:

● authentication and authorization;

● interaction with the mobile application through obtaining locations, their analysis and subsequent storage in the database;

● control of database queries;

● sending locations to the web application, where they can be observed by the operator.

The client part of the mobile application was implemented using React Native technology and the main task of this part of the software is to inform the server part about the appearance of air targets in the visibility of the user of the mobile device.

Results of the program

The operator part of the web application is available only after user authorization and is used by the operator of air defense systems. Two main functional components are implemented in the operational part of the software package. Data is a map that will later display the locations in which users have recorded air targets. People – a map that displays all users of the software package (to understand on which boundaries of the territory users are placed). Fig. 4 shows the interface of the operator chat of the software package.

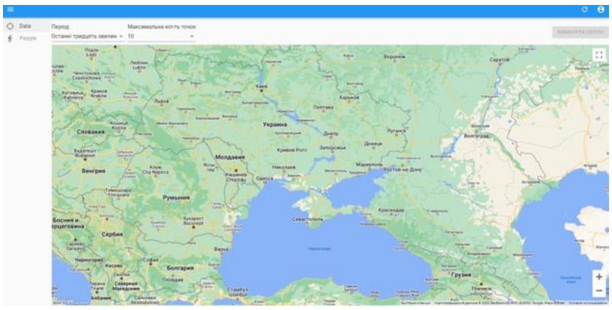

Figure 4 - Interface of the operator part of the software package

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Fig. 5 shows the operator's software interface for tracking the number of users involved in the operation of the alternative warning system. We emphasize that the information in Fig. 5 has been changed and does not reflect the real picture. The authors do not have reliable information, so the image in Fig. 5 is a model and reflects only the principles of operation of the corresponding program interface. The number indicated in the black circle reflects the number of users, with a decrease in the scale of the map and significant discrepancies in the location of users, the circles are divided and at a minimum scale become separate with a value of 1 indicated on them.

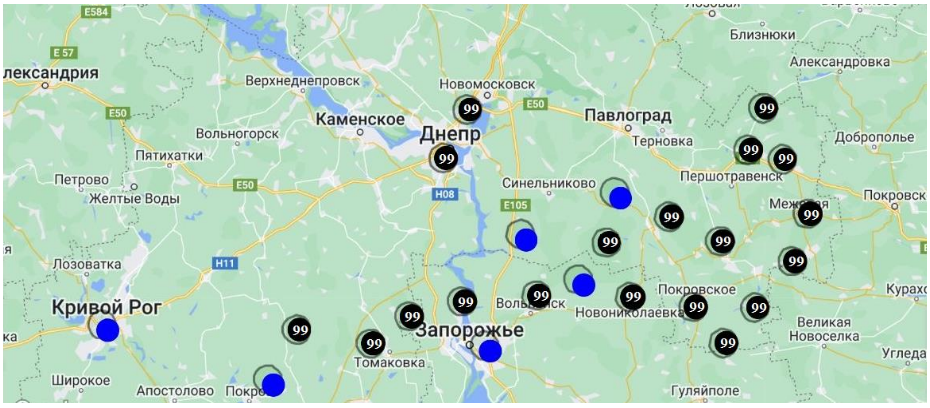

Figure 5 - Interface of the "People" tab of the operator part of the software package

Among the settings of the operator part of the software package, there are filters by the time period of displaying labels about the transferred locations (you can choose one of the options - the last thirty minutes, the last 3 hours, the last day) and a filter by the maximum number of points on the map (10, 25, 50 and 100), which allows you to work with different time periods and the number of observation points. The "Turn off the light" function is also implemented, which allows you to remove all points from the map. Such a functional component is used in massive shelling, when wave after wave reports are received about air targets in the same locations. Fig. 6 shows an example of three users informing the operator about the flight of an air target at different times.

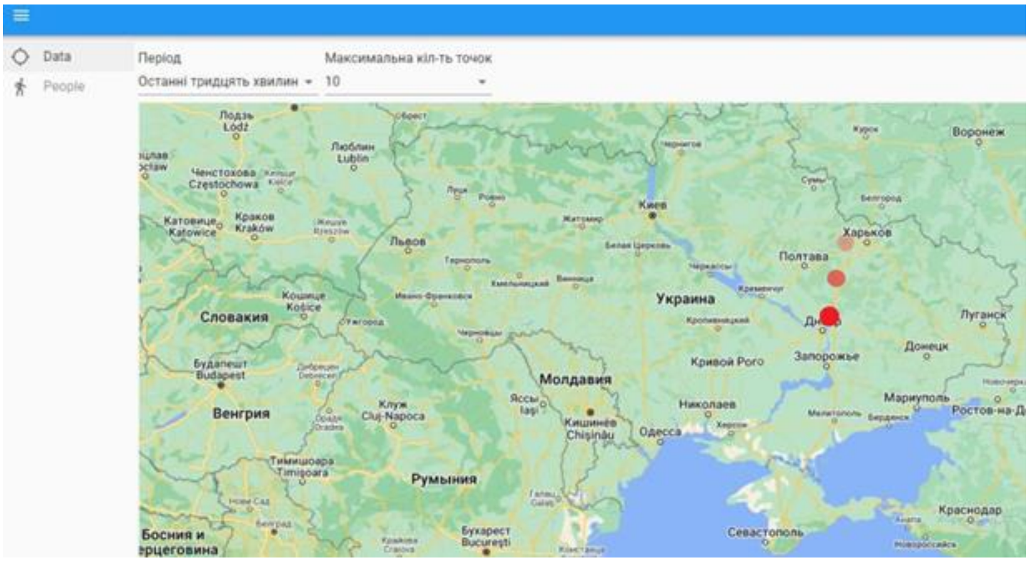

Figure 6 - Illustration of informing in the operator part of the software package

By changing the color intensity of the display of the locations of fixing air targets, the operator can understand the direction of flight of the missile, because the newest points will have a higher color intensity.

Since the map can be scaled, it is possible to display different territories, namely the entire country, region and even city (with lower-speed targets such as UAVs). Fig.7. an example of displaying a moving target within the city of Kamianske is given.

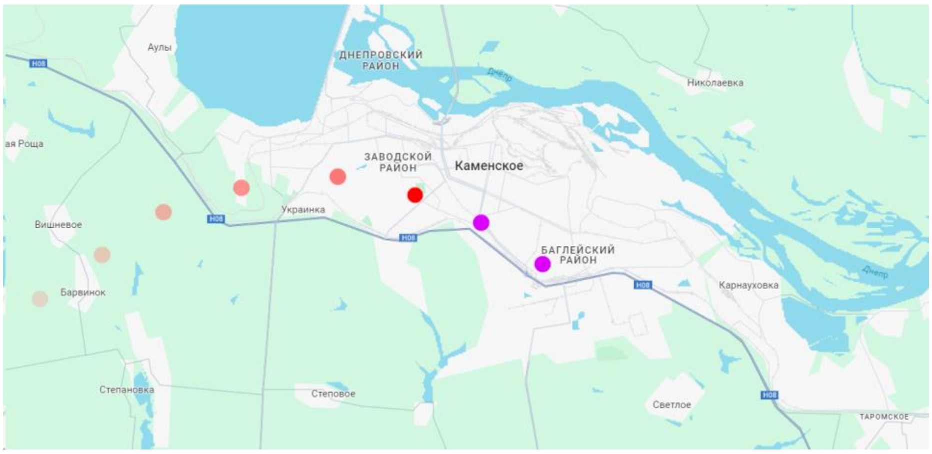

Figure 7 – Visualization of the trajectory of a moving air target within the city of Kamianske.

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Solving the problem of optimal placement of fire groups

The paper calculates six examples of solving problems of optimal placement of fire groups in the formulation of the problem described in this work earlier. As initial conditions (training sample). Table 1 presents data calculated for nine different time periods and a different number of mobile groups. We would like to emphasize that the number of fire groups is chosen at random and cannot be considered as reliable information, such information is secret and is not known to the authors.

It should be noted that numerical calculations were obtained on the example of the Dnipropetrovsk region, because it was for it that the largest amount of information about air targets and their routes was previously collected. The range of mobile groups is R=5 km, the region is chosen in the form of a rectangle, which fully accommodates the Dnipropetrovsk region, the sides of the rectangle touch its northern, southern, western and eastern extreme points, including parts of adjacent regions. Due to the large amount of information about air targets that were taken into account in each task (more than 20), such information in the table will be presented in the format X:YYY:ZZ. Where X – takes the value "N", "S", "E", "W" if the air target entered the specified area from the north, south, west and east, respectively. YYY the displacement of the entry point (the distance is indicated in kilometers) at the corresponding border (for the western and eastern directions of entry, the coordinates increase upwards, for the northern and southern directions of entry, the coordinates increase to the right), ZZZ is a degree measure of the angle of the direction of movement of the air target at the moment of entering the area, where 0^0 is up, 90^0 is up, 180^0 is left, 270^0 is down according to the principle of a "single circle".

Table 1

Results of the numerical solution of the problem (1)-(4) for the territory of the Dnipropetrovsk region

(yy/mm/dd - hh:mm)	Num.of mobile fire gr-ps	Characteristics of aerial targets	Junctions
22/10/04-12:04	6	S:53:112 N:63:209 E:32:-13 E:113:-85 S:24:141 N:89:304 N:110:224 E:10:-18 S:115:163 W:84:219 E:190:44 E:163:-76 N:13:207 N:36:230 E:111:24	(170;115), (196;187), (120;67), (113;25), (293;27), (90;46)
22/11/18-08:01	5	N:146:353 N:135:193 E:144:36 S:141:75 W:184:136 S:93:28 W:85:268 E:65:-63 W:79:150 N:110:213 W:0:101 W:0:185 N:45:352 S:108:41	(122;158), (97;21), (195;162), (180;191), (272;52)
22/12/16-14:12	8	W:9:122 W:11:217 W:56:127 N:124:305 N:95:335 E:58:-79 E:10:-75 N:145:223 N:138:259 S:100:118 E:133:47 W:198:138 N:138:327 W:159:245 S:28:34 S:9:14 S:10:121 W:175:260	(196;27), (40;175), (42;44), (108;77), (174;121), (174;106), (217;173), (89;111),
23/05/12-03:12	6	W:102:233 S:53:131 W:167:96 E:174:10 S:30:36 W:161:182 N:143:241 S:38:114 E:51:-8 E:196:85 N:3:254 W:18:26 W:153:111 S:119:134 N:133:352	(103;2), (99;189), (242;29), (37;197), (43;130), (189;34)
23/06/21-23:05	6	W:92:129 W:6:168 S:115:10 N:128:184 N:71:223 N:50:214 S:54:151 S:26:74 S:50:166 N:132:181 W:82:142 S:20:72 W:17:142 E:169:52 S:11:99	(185;47), (187;105), (90;35), (277;90), (62;165), (268;54)
23/08/03-16:13	4	S:58:34 E:3:32 W:179:257 S:47:128 S:80:110 N:117:315 E:36:17 S:92:70 W:129:198 E:65:69 E:43:13 E:142:-82 N:17:331 S:72:100 N:107:256	(149;152), (204;147), (145;199), (163;113)
24/01/08-18:00	6	S:110:81 E:94:-20 E:59:-67 E:139:77 S:43:153 S:42:164 N:39:322 E:199:72 E:167:-14 E:147:29 W:160:229 N:50:356 S:48:142 E:47:51 N:13:221	(9;153), (231;14), (82;168), (160;198), (125;155), (171;54)
24/03/14-16:42	4	S:127:142 S:46:51 W:180:184 W:88:197 W:1:100 N:18:183 W:171:176 W:46:132 S:77:40 E:138:82 N:78:238 S:124:33 S:29:172 W:14:195 N:86:244	(165;122), (222;160), (146;114), (118;11)
24/08/20-10:19	6	E:111:24 S:11:99 S:109:36 E:101:-4 W:46:254 E:58:71 N:138:352 E:121:-60 W:133:105 W:189:129 E:103:49 N:146:329 S:55:83 S:72:153	(277;182), (217;64), (80;143), (62;133), (289;104), (154;40)

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Analysis of the results obtained

Table 1 shows the results of numerical calculations of the problem (1)-(4) with different input data. As we can see, the calculated location of mobile fire groups with their different numbers and different characteristics of air targets has a fairly uniform structure, but differs from the uniform distribution precisely due to taking into account the information about the direction and trajectory of air targets, which are taken into account in the calculation. We would like to emphasize that the number of air targets used in the calculations is not limited and can/should differ significantly from the model ones upwards. Therefore, the results of numerical experiments can be considered reliable, and the model can be taken as a basis for the development of recommendations for the location of mobile fire groups.

Conclusions

The paper formulates and solves the problem of optimal placement of mobile fire groups and develops a mathematical model of the alternative warning system "Light", which was successfully used in the Dnipropetrovsk region from March 2022 to January 2023. A software package with a user and operator part was implemented, a database and a management system were developed. The solved task takes into account information about enemy air targets that were recorded by the alternative warning system "Light" and forms recommendations regarding the location of a given number of mobile fire groups.

Taking into account the substantive applied part of the development and the clearly defined interpretation of the parameters of the model and the system, we see further directions of research in this direction, in particular:

• solving a modified problem with restrictions on repeating configurations for the placement of mobile fire groups;

• solving a modified problem using various metrics, in particular with reference to the roads of the region under study;

development of a software package for visualizing solutions to the problem with the possibility of using an algorithm in the presence of external control of the operator.

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О.О. Кузенков

АЛГОРИТМИ ТА МЕТОДИ В ДИНАМІЧНИХ ЗАДАЧАХ ОПТИМАЛЬНОГО РОЗМІЩЕННЯ ВОГНЕВИХ ГРУП

Анотація. *Робота присвячена розвязанню актуальної задачі розгортання вогневих угруповань сил ППО для прикриття рубежів повітряного нападу. Для вирішення поставленої задачі використовуються сучасні методи теорії оптимального розбиття множин, математичного моделювання з використанням диференціальних рівнянь та їх систем, розроблено спеціалізований програмний пакет, що включає мобільний та браузерний додаток. При розробці програмного комплексу використовувалися сучасні мови та технології програмування, проводився огляд та порівняння з існуючими рішеннями в різних країнах. Проведено ретроспективний аналіз, розроблено критерії оптимальності шуканого рішення, визначено переваги та недоліки підходу.*

Використання методів теорії оптимального розбиття множин дозволяє аналітично визначати критерії якості розв'язку, проводити аналітичне дослідження допустимих розв'язків та оцінити оптимальність рішення, а апробація аналітичних результатів на практиці та прикладах з реальними *вхідними даними підвищує точність та релевантність результатів, отриманих аналітично.*

На сьогодні питання динамічного розгортання вогневих груп є дуже актуальним для України. Рішення про створення та функціонування вогневих груп було прийнято та реалізовано лише з початку повномасштабного вторгнення країни-агресора та набуло широкого застосування під час масованого застосування країною-агресором ударних безпілотників проти цивільних, військових та енергетичних об’єктів України. Ефективність використання вогневих груп доведена практикою, але спосіб їх розміщення, як правило, є евристичним, тобто не оптимальним.

Ключові слова: *оптимальне розбиття множин, динамічна задача, вогнева група, ППО, програмний комплекс.*

Oleksandr O. Kuzenkov – Candidate of Physical and Mathematical Sciences, Associate Professor, Associate Professor of the Department of Computational Mathematics and Mathematical Cybernetics, Faculty of Applied Mathematics and Information Technologies, Dnipro National University named after Oles Honchar, 72 Nauky Avenue, Dnipro, 49000, Ukraine. ORCID: https://orcid.org/0000-0002-6378-7993, kuzenkov1986@gmail.com, Scopus Author ID: 56154949000

Кузенков Олександр Олександрович – кандидат фізико-математичних наук, доцент, доцент кафедри обчислювальної математики та математичної кібернетики, факультету прикладної математики та інформаційнизх технологій Дніпровського національного університету імені Олеся Гончара, 72 проспект Науки, Дніпро, 49000, Україна.