DOI: 10.34185/1991-7848.itmm.2024.01.048

AUTOENCODER NEURAL NETWORK FOR UNIVARIATE TIME SERIES EMBEDDING

Eugene Koshel

Department of computer technologies, Faculty of applied math, Oles Honchar Dnipro National University, Ukraine

Abstract. *The problem of time series embedding is a universal one. It is the main prerequisite when it comes to modeling of dynamical processes using systems of autonomous ordinary differential equations (ODEs) because they have hard requirements for the dimensionality of the problem. One-dimensional ODE can only exhibit 3 types of behavior while two-dimensional ODE can exhibit 9. This is why it is important to increase the dimensionality of the problem before starting the modeling to allow for wider range of possible behaviors in the final model. One way to increase the dimensionality is to delay-embed the time series data but this approach can be extended to allow the use of an autoencoder neural network that would associate a higher-dimensional vector to each point in the time series and will allow the modeling to be performed in higher dimension.*

Keywords: *time series analysis, time series embedding, dimensionality reduction, autoencoder, neural network, time series modeling, Takens's theorem.*

Introduction. Time series analysis is the process of extracting information from the series of data points in an attempt to obtain useful information about the system that produced it or to predict the next values in the series. The simplest form of the time series is a univariate series or just a sequence of one-dimensional measurements. If the system that the data was obtained from is sufficiently complex, the time series may exhibit periodic, quasi-periodic, chaotic behavior, drift, etc. It is easy for a human to see whether these behaviors are present or not simply by looking at the plotted sequence of data points, but this can hardly qualify as proper analysis. Luckily, there are methods developed in different fields that can be used for identification and quantification of different properties of the time series data. Most of these methods, however, have a hard requirements for the dimensionality of the problem and can yield nonsensical results if the dimension of the problem they are applied to is too low or too high. This issue can be solved by embedding the available time series data into a higher-dimensional space where the analysis methods can be applied.

Time series embedding is a mapping between the segments of the time series data and some vector space R^n. This mapping is expected to be an injective function that produces a smooth trajectory of embedding vectors for adjacent time series data segments. Usually the mapping is an unknown function that has to be approximated. Currently, the most popular method for approximating unknown functions is fitting a neural network to the data so that it will learn the hidden dependencies and will be able to produce accurate results for new inputs it was not trained on.

Research results. Consider the following formulation of the Takens's theorem [0]:

$$\varphi(X)=\left(\alpha(X),\alpha(f(X)),\ldots,\alpha(f^{k-1}(X))\right),\quad \alpha(f^i(X))=x_{t-\tau_i}, \tag{1}$$

It states that there exists a function α that, when applied to the true vectors on the attractor of the modeled system, yields the time series data, separated from each other in time. From (1) it follows that if a segment of the time series is multiplied by a matrix of shape:

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1d} \\ a_{21} & a_{22} & \cdots & a_{2d} \\ \vdots & \vdots & \vdots & \vdots \\ a_{\tau_{max}1} & a_{\tau_{max}2} & \cdots & a_{\tau_{max}d} \end{pmatrix}, \tag{2}$$

where columns of A are unitary vectors and $\forall j \exists i: a_{ij}=1$. The number of columns d is the embedding dimension and the number of rows is given by the largest selected delay value. But if we also assume the following in (1):

$$\alpha(f^i(X))=\beta(x_t,x_{t-1},\ldots,x_{t-l}) \tag{3}$$

where $\beta:\mathrm{R}^l \to R$ is the *feature extraction function* that maps time series segments to the codomain of the unknown observation function α. The matrix (2) can be replaced with any function that maps the segment of the time series to the embedding space. For example, the Broomhead-King approach derives the matrix by using SVD decomposition of the time series [0] and the short-term Fourier transform utilizes a complex-valued matrix [0] for embeddings. In our case, we use an autoencoder network that models both α and β functions simultaneously as its encoder and decoder components:

$$E(x)=A_e x+b_e\,,\qquad D(y)=A_d y+b_d\,, \tag{4}$$

where A_e is a N by m matrix that maps N points of the trajectory into the m dimensional latent space, . A_d . is a m by 1 matrix that maps the points in the latent space to the last point of the converted segment of the time series.

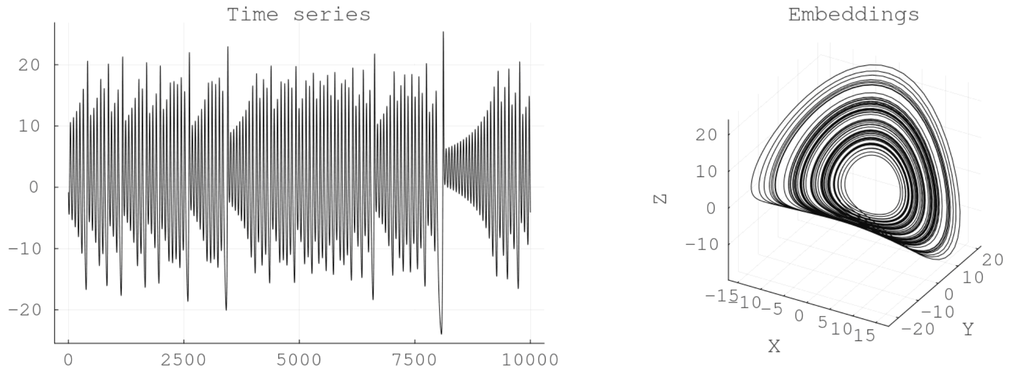

Figure 1: The embedding of the test time series obtained from calculting the norms of the vectors in the Lorenz system trajectory and balancing them to have a mean of zero.

The resulting embeddings are provided on the Fig. 1.

Conclusion. We have shown that valid time series embedding can be achieved by using an autoencoder neural network. The test embedding satisfies the Takens's theorem and accurately preserves the qualitative characteristics of the original system.

ЛІТЕРАТУРА / REFERENCE

1. Broomhead D.S. and King G. P. (1986). Extracting qualitative dynamics from experimental data. Physica D: Nonlinear Phenomena, Volume 20 (2-3), Pages 217-236. https://doi.org/10.1016/0167-2789(86)90031-X.

2. Gupta, V. and Mittal, M. (2019). QRS Complex Detection Using STFT, Chaos Analysis, and PCA in Standard and Real-Time ECG Databases. J. Inst. Eng. India Ser. B 100, 489–497. https://doi.org/10.1007/s40031-019-00398-9

3. Takens, F. (1981). Detecting strange attractors in turbulence. In: Rand, D., Young, LS. (eds) Dynamical Systems and Turbulence, Warwick 1980. Lecture Notes in Mathematics, vol 898. Springer, Berlin, Heidelberg. https://doi.org/10.1007/BFb0091924

НЕЙРОННА МЕРЕЖА ТИПУ АВТОКОДУВАЛЬНИК ДЛЯ ВКЛАДЕННЯ ОДНОВИМІРНИХ ЧАСОВИХ РЯДІВ

Кошель Є.В

• *Кафедра комп'ютерних технологій, Факультет прикладної математики, ДНУ ім. О. Гончара, Україна*

Анотація. *Задача вкладення часових рядів є доволі універсальною. Вона являє собою перший крок у процесі моделювання динамічних процесів за допомогою автономних систем диференціальних рівнянь, оскільки такі системи накладають суворі обмеження на розмірність модельованих процесів. Одновимірні диференціальні рівняння мають всього 3 можливі типи поведінки, у той час як двовимірні мають 9. Саме тому важливо збільшувати розмірності задачі перед початком моделювання, щоб дозволити фінальній моделі мати ширший діапазон поведінок. Одним зі способів збільшення розмірності є вкладання з затримкою, але його може бути розширено та узагальнено, якщо використати нейронну мережу-автокодувальник, яка б побудувала залежність між сегментами часового ряду та високорозмірними векторами в просторі вкладення.*

Ключові слова: *аналіз часових рядів, вкладення часових рядів, зміна розмірності, автокодувальник, нейронна мережа, моделювання часових рядів, теорема Такенса.*